Scalar Mesons, Glueballs, Instantons and the Glueball/Sigma

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Abstract

We include instanton effects in QCD sum rules for coupled scalar glueballs and mesons. We find a light glueball/sigma as in earlier studies without instantons, but in a lattice-type pure instanton model the light glueball/sigma is not found. In the 1-2 Gev region we now find that lightest I=0 meson, in the region of the $f_o(1370)$, has no direct glueball mixing, with the instanton loop replacing the glueball component. The lightest scalar mainly glueball in the region of the $f_o(1500)$ is sensitive to the choice of nonperturbative gluonic parameters.

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1 Introduction

Some of the earliest applications of the method of QCD sum rules was for the study of gluonic hadrons[1], known as glueballs. Using the sum rules obtained in that work it was observed that a light scalar glueball solution in the region of 400-500 MeV [2] might be strongly coupled to a $\pi\pi$ resonance we call the sigma, which would give large branching ratios for the decays into channels with sigmas of heavier glueballs and hybrids[3]. Moreover, it was observed[2] that if the coupling of scalar mesons to glueballs is included, the original scalar meson QCD sum rules[4] are modified so that the lightest 80% meson solution predicts a mass about 400 MeV higher than the pure $q\bar{q}$ solution, i.e., near the $f_o(1370)$ rather than the $f_o(980)$ as found in the earlier work[4]

It has been known for decades that instantons can represent a large part of nonperturbative gluonic interactions. In the present work we include instanton effects in the sum rules for the scalar glueballs and mixed scalar mesons and glueballs. In Sect. 2.1 we review the QCD sum rules for scalar mesons, glueballs and mixed meson-glueballs without instantons. In Sect. 2.2, we review work that has been done on scalar hadrons with instantons using the sum rule methods and give the modification of the sum rules with instantons included. In Sect. 3 we give the results for a possible light glueball/sigma of mass about 500 MeV and for mixed meson-glueballs in the 1-2 GeV region. If we drop all gluonic condensates from the sum rules, but retain the instantons as the source of nonperturbative effects to agree with recent quenched lattice calculations, we no longer find the light glueball. Instead, we find only scalar glueball solutions with masses greater than 1400 MeV. We discuss the results in Sect. 4.

2 Sum Rules for Mixed Scalar Mesons and Glueballs

In this section we discuss the QCD sum rules for mixed scalar mesons and quarks. The method[5] makes use of a correlator defined in terms of a composite field operator

$$\Pi(p) \;=\; i\int d^4x\, e^{iq\cdot x} < 0 \mid T[J(x)J(0)] \mid 0 > , \tag{1}$$

where J(x) is a field operator composed of quark and/or gluon fields for pure QCD. In the present work the operators are

$$\begin{aligned} J^m(x) &= \frac{1}{2}(\bar{\mathrm{u}}(\mathrm{x})\mathrm{u}(\mathrm{x}) - \bar{\mathrm{d}}(\mathrm{x})\mathrm{d}(\mathrm{x})) \\ J^G(x) &= \alpha_s G^2, \end{aligned} \tag{2}$$

for the I=0 scalar meson and glueball, respectively. The I=1 scalar meson is not treated here. The QCD sum rules are obtained by equating a dispersion relation for the correlator to a QCD evaluation using an operator product expansion (OPE). We discuss these sum rules

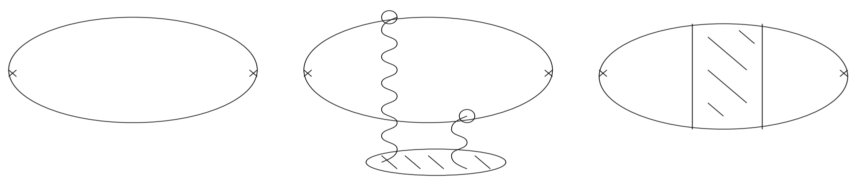

Fig. 1 a) perturbative loop b) gluon condensate c) four-quark condensate

in the following two subsections, first without explicit instantons and secondly with explicit instanton contributions.

2.1 Mixed Scalar Mesons and Glueballs Without Instantons

In this subsection we review QCD sum rules for scalar mesons, for scalar glueballs, and for the mixed meson-glueball sum rules. As was discussed in the first QCD sum rule research on scalar glueballs[1], there is a strong argument for using a subtracted dispersion relations, while the first work on scalar mesons[4] used an unsubtracted dispersion relation. This is important consideration for the present work.

2.1.1 Scalar mesons

The QCD sum rules for scalar mesons were first treated in Ref. [4]. The most important processes, obtained by an OPE, are illustrated in Fig. 1 for light-quark mesons, and in Euclidean momentum space (Q^2=-p^2) give

$$\begin{aligned} \Pi(Q)^{QCD} &= \frac{3}{8\pi^2}(1+\frac{11}{3}\frac{\alpha_s}{\pi})Q^2 ln(Q^2) \\ &+\frac{\alpha_s}{8\pi Q^2} < \mathrm{G}^2 > +\frac{\pi\alpha_s}{Q^4}P^{4q}, \end{aligned} \tag{3}$$

where $< \mathrm{G}^2 >$ is the gluon condensate and P^{4q} is the four-quark condensate for the scalar meson, illustrated in Fig. 1c. The perturbative gluon correction diagram is not shown. Note that the quark condensate term, $< \bar{\mathrm{q}}\mathrm{q} >$, is neglected as it is proportional to the current quark mass. The factorized form For the four-quark condensate [5], $P^{4q} \simeq -176 < \bar{\mathrm{q}}\mathrm{q} >^2 /27$, is probably accurate to about a factor of two. After the Borel transform the sum rule, with M the Borel mass, is

$$\begin{aligned} g_0 e^{-M_m^2/M^2} &= \frac{3}{8\pi^2}(1+\frac{11\alpha_s}{3\pi})M^4 E_1(s_o/M^2) \\ &+\frac{\alpha_s}{8\pi} < \mathrm{G}^2 > -\frac{176\alpha_s}{27M^2}c_4 < \bar{\mathrm{q}}\mathrm{q} >^2, \end{aligned} \tag{4}$$

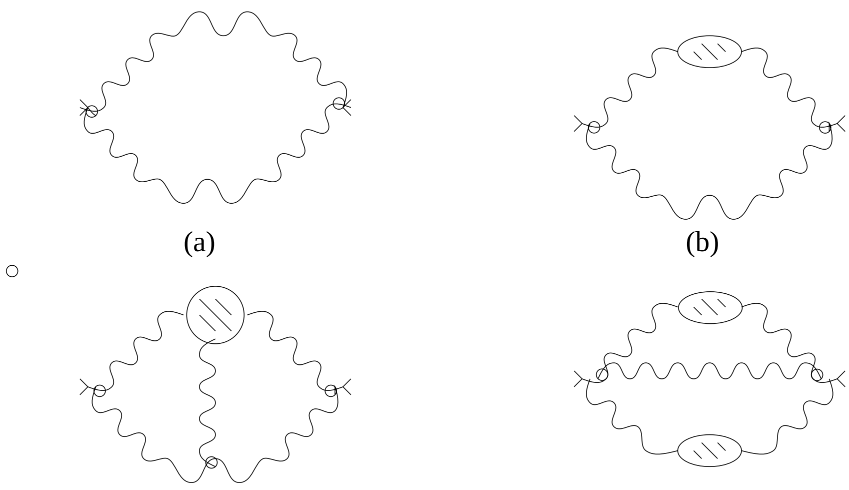

Fig. 2 Glueball processes: a) perturbative loop, b) gluon condensate
c) D=6 gluonic condensate, $\Gamma^{(6)}$, d) D=8 gluonic condensate, $\Gamma^{(8)}$

with c_4 a constant representing the correction to the four-quark condensate, and g_0 a D = 4 constant which will not be used in the analysis. The functions $E_n(s_0/M^2)$ represent the continuum contribution to the sum rules with a simple form for the continuum spectral function. They are defined as $E_n(x) = 1 - e^{-x}\sum_{k=0}^{n}\frac{x^k}{k!})$

Taking the derivative of Eq.(4) with respect to $1/M^2$ and using the ratio of the resulting equation and Eq.(4 one obtains an equation for the scalar meson mass:

$$M_m^2 = \frac{\frac{3}{8\pi^2}(1+\frac{11\alpha_s}{3\pi})(2M^6E_1(s_0/M^2) - s_o^2M^2e^{-\frac{s_o}{M^2}}) - \pi\alpha_s P^{4q}}{\frac{3}{8\pi^2}(1+\frac{11\alpha_s}{3\pi})M^4E_1(s_0/M^2) + \frac{\alpha_s}{8\pi} < \mathrm{G}^2 > + \frac{\pi\alpha_s}{M^2}P^{4q}} \tag{5}$$

There is a stable solution for the scalar meson mass M_m of about 1 GeV, which is the result of the original work of Ref.[4]. If this were the physical solution then we would interpret this as the $f_0(980)$, however, it was found in previous work [2] that the coupling to the scalar glueball increases the mass by 3-400 MeV. In the simple picture of Eq.(5) the I=1 is degenerate with the I=0 meson, giving the $a_0(980)$.

2.1.2 Scalar glueballs

The QCD sum rules for scalar glueballs were first treated in Ref. [1]. There have been many calculations of scalar glueballs in recent years [6, 2]. With the preturbative corrections of Ref.[7] and including nonperturbative terms up to dimension eight one finds for the correlator corresponding to the processes illustrated in Fig. 2

$$\begin{aligned}\Pi(Q^2)^{QCD} &= -2(\frac{\alpha_s}{\pi})^2(1+\frac{51}{4}\frac{\alpha_s}{\pi} - \frac{11}{4}\frac{\alpha_s}{\pi}ln(Q^2))Q^4ln(Q^2) + 4\alpha_s^2 < \mathrm{G}^2 > (1+ \\ &\quad \frac{49}{12}\frac{\alpha_s}{\pi} - \frac{11}{4}\frac{\alpha_s}{\pi}ln(Q^2)) + \frac{8\alpha_s^2\Gamma^{(6)}}{Q^2}(1-\frac{29}{4}\alpha_s ln(Q^2)) + \frac{8\pi\alpha_s^3\Gamma^{(8)}}{Q^4},\end{aligned} \tag{6}$$

where $\Gamma^{(6)} = \langle g_s f_{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}\rangle$ and $\Gamma^{(8)} = \langle 14(f_{abc} G^a_{\mu\nu} G^b_{\nu\rho})^2 - (f_{abc} G^a_{\mu\nu} G^b_{\rho\lambda})^2\rangle$ are dimension 6 and dimension 8 gluonic condensates, illustrated in Figs 2c) and d), respectively. Note that the largest nonperturbative contribution from the gluon condensate is independent of momentum, since the dimension of $< \mathrm{G}^2 >$ is the same as the correlator. For this reason it does not contribute to the correlator after the Borel transform. For this reason it was suggested [1] that one should use a subtracted dispersion relation. Using the subtracted form, $(\Pi(Q^2) - \Pi(0))/Q^2$ and taking the Borel transform one obtains the sum rule

$$
\begin{aligned}
\Pi(0)e^{-M_G^2/M^2} + cont. \;&=\; \Pi(0) + 2(\frac{\alpha_s}{\pi})^2(1 + \frac{51}{4}\frac{\alpha_s}{\pi} - \frac{11}{2}\frac{\alpha_s}{\pi}(1 - \gamma_E + ln(M^2))M^4 \\
&\quad E_1(s_o/M^2) - 4\alpha_s^2 < \mathrm{G}^2 > (1 + \frac{49}{12}\frac{\alpha_s}{\pi} + \frac{11}{4}\frac{\alpha_s}{\pi}(\gamma_E - ln(M^2)) \\
&\quad -\frac{8\alpha_s^2\Gamma^{(6)}}{M^2}(1 - \frac{29}{4}\alpha_s(1 - \gamma_E + ln(M^2))) - \frac{8\pi\alpha_s^3\Gamma^{(8)}}{2M^4}.
\end{aligned} \tag{7}
$$

Taking the ratio of the $\partial/\partial(1/M^2)$ of Eq.(7) to Eq.(7) one has a sum rule for a pure scalar glueball, with no explicit quark/antiquark components, although the qluonic condensates contain important quark pair contributions,

$$
\begin{aligned}
M_G^2 \;=\;& [2(\frac{\alpha_s}{\pi})^2(2(1 + \frac{51}{4}\frac{\alpha_s}{\pi} - \frac{11}{2}\frac{\alpha_s}{\pi}(1 - \gamma_E + ln(M^2)))M^6 E_2(s_o/M^2) \\
& -\frac{11}{2}(\frac{\alpha_s}{\pi})M^6 E_1(s_o/M^2)) + 11(\frac{\alpha_s^3}{\pi}) < \mathrm{G}^2 > M^2 + 8\alpha_s^2\Gamma^{(6)} \\
& (1 + \frac{29}{4}\alpha_s(\gamma_E - ln(M^2)) + \frac{8\pi\alpha_s^3}{M^2}\Gamma^{(8)}] \\
& [\Pi(0) + 2(\frac{\alpha_s}{\pi})^2(1 + \frac{51}{4}\frac{\alpha_s}{\pi} - \frac{11}{2}\frac{\alpha_s}{\pi}(1 - \gamma_E + ln(M^2)))M^4 E_1(s_o/M^2) \\
& -4\alpha_s^2 < \mathrm{G}^2 > (1 + \frac{49}{12}\frac{\alpha_s}{\pi} + \frac{11}{4}\frac{\alpha_s}{\pi}(\gamma_E - ln(M^2)) \\
& -\frac{8\alpha_s^2}{M^2}\Gamma^{(6)}(1 - \frac{29}{4}\alpha_s(1 - \gamma_E + ln(M^2))) - \frac{4\pi\alpha_s^3}{M^4}\Gamma^{(8)}]^{-1}.
\end{aligned} \tag{8}
$$

Using the range of $\Gamma^{(6)}$ and $\Gamma^{(8)}$ suggested in Refs.[6, 7] one finds stable solutions for a light glueball with a mass in therange of 300 MeV-600 MeV. This is the glueball/sigma of Ref. [2], which has been used for calculating the sigma branching ratio from hybrid decay[3] and estimates of diffractive sigma production in high energy proton-proton processes mediated by the Pomeron[8].

2.1.3 Mixed scalar glueballs and mesons

Physically it is expected that scalar glueballs and mesons should mix, and from the early calculations using QCD sum rules for scalar hadrons[1, 9] this was discussed. This suggests that one must use a scalar current of the form

$$
J_{0^{++}} \;=\; \beta M_o J_m + (1-\mid \beta \mid)J_G, \tag{9}
$$

where M_o is a constant that we take to be 1 GeV. The mechanism for mixing is given by a low energy theorem discussed in Ref. [1]. One can show that the mixing term in the correlator with the current of Eq.(9) is [2]

$$\Pi^{mixing} \simeq \beta(1- \mid \beta \mid)\frac{64}{9} < \bar{\mathrm{q}}\mathrm{q} > . \tag{10}$$

Since the Borel transforn of a constant vanishes, the mixing term does not contribute to the unsubtracted sum rule, while it contributes the term shown in Eq.(10) to the subtracted dispersion relation. One finds the solutions to the sum rules discussed in Ref. [2], where it was reported that there are no stable solutions in the 1-2 GeV region without glueball-meson mixing. The most stable solutions were found for a 80% meson and an 80% glueball. These presumably correspond to the $f_o(1370)$ and $f_o(1500)$, respectively, although the uncertainty of about 15% in the solutions cannot separate these two solutions. Note that there is no solution near the $f_o(980)$, which shows the importance of the gluonic mixing (recall that without the mixing the lowest mesonic solution is near the $f_o(980)$[4]). The purely gluonic solution in the 300-600 MeV range is a prediction of the method. In the next sections we study the solutions to the sum rules with instanton effects included.

2.2 Mixed Scalar Mesons and Glueballs With Instantons

For many years it has been known that instantons can represent a major part of the nonperturbative gluoic interactions. See Ref. [12] for an excellent review of the concepts of instantons and applications to QCD. The starting point is the solution for the instanton using the classical action [10], which gives for the instanton color field

$$\begin{aligned} A^{inst}_{\mu(x)a} &= \frac{2\eta_{a\mu\nu}x_\nu}{x^2+\rho^2} \\ G^{inst}(x)\cdot G^{inst}(x) &= \frac{192\rho^4}{(x^2+\rho^2)^4} \end{aligned} \tag{11}$$

where ρ is the instanton size. From this the quark zero modes were derived [11], which is the main basis for subsequent research with quarks and instantons. Although in the instanton gas picture, with $\rho \simeq 1.0$ fm[1], the inclusion of instantons in QCD sum rules did not seem very promising for hadronic physics, in the instanton liquid model [13] with $\rho \simeq 1/3$ fm instantons give large nonperturbative effects in the medium-range region, where neither the perturbative glue nor the long range confining glue represented by the condensates are effective. On the other hand the other hand the instantons cannot give confinement (see Ref.[12] for a discussion and references). From this we conclude that one needs both instanton and gluonic condensate processes in the QCD sum rules for scalar hadrons.

In a previous study of the role of instantons in hadronic physics the solution for a quark in the instanton-antiinstanton medium [14] was tested using the Dyson-Schwinger (DS) equation [15] with a confining gluonic propagator that was fit to the condensates, and it was found that there is no consistency with the condensates. An important observation for the present work is that although the instantons can produce most of the quark condensate (dimension = 3) they do not give the correct higher dimensional mixed condensate (dimension = 5). This can be explained by the higher dimensional condensates produced by gluonic effects at a larger length scale than the 1/3 Fm of the instanton liquid model. Recently, the DS equation was solved both on the light-cone [16] and in four-dimensional Euclidean space [17], and consistent solutions are obtained if both instantons and a confining gluonic propagator are included. Moreover, the instantons provide the largest nonperturbative effects.

In the present work we assume that there are three length scales: the perturbative region with L$\leq$ 0.2 fm, the midrange nonperturbative region with L$\simeq$ 0.33 fm given by instantons and the confining region with L$\geq$ 0.5 fm. Our model is as follows:

1) The color field can be written as A = $A^{inst} + \bar{A}$, with the instanton being the classical solution and $\bar{A}$ the residual color quantum field.

2) The instanton loop is included in the correlator for scalar glueballs, but no instanton interactions are included. The latter are assumed to be accounted for by the various condensates. This is consistent with the instanton liquid model [12], in which the paramenters are constrained by the gluon condensate.

3) In the meson correlator the loop of quark-antiquarks in the background instanton medium is included, but no instanton interacton processes, for similar reasons.

Thereby, we add the instanton processes to the perturbative processes for the sum rules. This adds two processes to the QCD side of the correlator. For the scalar meson correlator the additional process is the loop with a quark and an antiquark in the instanton medium. After a Borel transform the instanton contribution to an isospin = I scalar meson correlator is [18, 19, 20]

$$\Pi^{\mathrm{q\bar{q},inst}} = (-1)^I \frac{3}{8\pi^2}\rho^2 M^6 e^{-x}(K_o(x) + K_1(x)), \tag{12}$$

with x=$\rho^2 M^2/2$. The instanton continuum contribution corresponding this is

$$\Pi^{\mathrm{q\bar{q},inst,cont}} = (-1)^I \frac{3}{4\pi}\int_{s_o}^{\infty} dss J_1(\rho\sqrt{s})Y_1(\rho\sqrt{s})e^{-s/M^2}, \tag{13}$$

with s_o the continuum parameter. The notation for the various Bessel functions is standard [21].

The new contribution of the instanton loop for the scalar glueball for the unsubtracted correlator, such as that used in Eq.(7), is (after the Borel transform) [1, 13]

$$\Pi^{GB,inst}(M) = -2^7\pi^2 n\frac{x^2}{\rho^2}e^{-x}(2x^3K_o(x) + (x^2+2x^3)K_1(x)), \tag{14}$$

where n is the instanton density. The corresponding continuum contribution is

$$\Pi^{GB,inst,cont}(M) = 2^4\pi^3 n\rho^4\int_{s_o}^{\infty} dss^2 J_1(\rho\sqrt{s})Y_1(\rho\sqrt{s})e^{-s/M^2}. \tag{15}$$

For the subtracted dispersion relationship, which gave the solution for the light glueball/sigma discussed above, the QCD and continuum contributions are

$$\Pi^{GB,inst}(M) \;=\; 2^6\pi^2 n x^2 e^{-x}((1+x)K_o(x)+(2+x+\frac{2}{x})K_1(x)) - 2^7\pi^2 n \tag{16}$$

for the instanton loop and

$$\Pi^{GB,inst,cont}(M) \;=\; 2^4\pi^3 n\rho^4 \int_{s_o}^{\infty} ds s J_1(\rho\sqrt{s})Y_1(\rho\sqrt{s})e^{-s/M^2}. \tag{17}$$

from the continuum.

3 Results

3.1 Light Scalar Glueball, Lattice Gauge Comparison

We now search for a solution to the subtracted correlator sum rules, which in the previous work without instantons [2] yielded the light scalar glueball/sigma. The sum rule solution is given by Eq.(8) with the addition of the instanton terms given by Eqs.(16,17), where the suitable derivative of the instanton terms must be carried out for the numerator of the equation. A crucial question is that with the inclusion of instantons how should one modify the values of the dimennsion 4, 6, and 8 gluonic condensates, $< G^2 >, \Gamma^{(6)}$, and $\Gamma^{(8)}$, discussed above. An analysis of recent QCD lattice calculations [22] finds that these quenched calculations give results similar to the instanton liquid model. However, this instanton model does not give the correct string tension. The interpretation must be that the instantons can give the quark condensate, where the length scale is about 1/3 fm, but cannot give the infrared nonperturbative QCD (NPQCD) effects. We explore this in the following manner: first we look for solutions with instantons and the gluonic condensates, studying the solutions with suitable choices of the gluonic condensates. We then look for solutions with the only NPQCD effects being those given by instantons, which should resemble the lattice gauge calculations [22].

We do indeed find solutions to the subtracted dispersion relation using higher-dimension gluonic condensates in the range previously found and with the standard parameters of the instanton liquid model [12]. Specifically, the size of the instanton is 1.67 GeV^{-1} and the instanton density is .0008 GeV^4. A typical solution as a function of the Borel mass is shown in Fig. 3. The greatest uncertainty in the method is the values of the gluonic condensates, particularly the higher-dimensional condensates, $\Gamma^{(6)}$ and $\Gamma^{(8)}$. The most widely accepted values of these higher dimensional gluonic condensates are $\Gamma^{(6)}$ = .0114 GeV^6 and $\Gamma^{(8)}$ = .0081 GeV^8. The solution shown in Fig. 3 has values of these condensates reduced by 20%. We also find stable solutions that meet the criteria of QCD sum rules in the range 400-600 MeV for values of the higher-dimensional condensates 40% to 100% of the accepted values mentioned. Fortunately, the ordinary gluonic condensate does not play a major role in these calculations once the instantons are included.

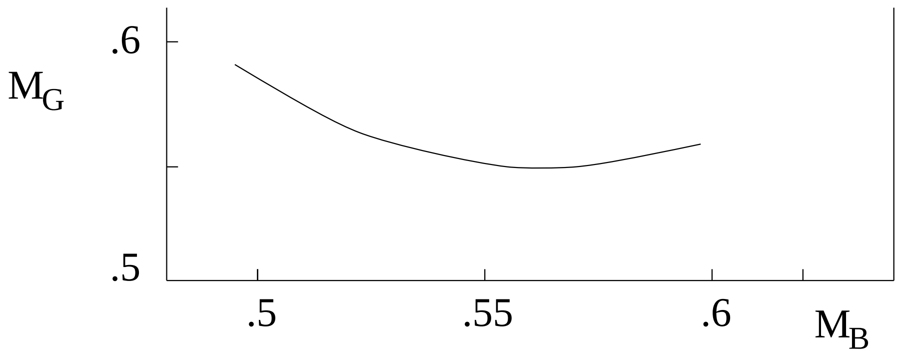

Fig. 3 Light glueball mass, M_G , vs Borel mass

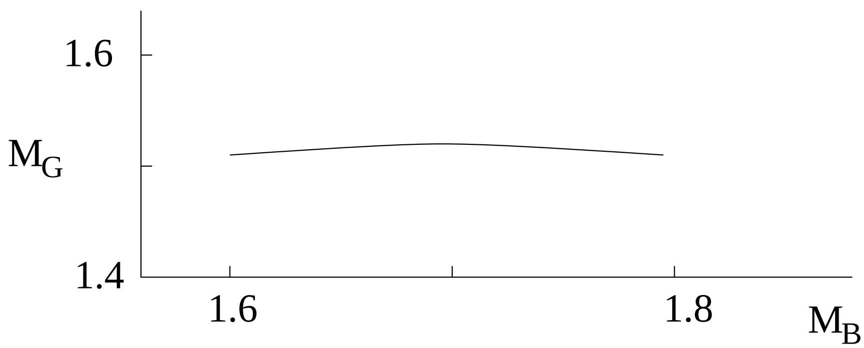

Fig. 4 Glueball mass, M_G , no gluonic condensates

Next we carry out studies of the subtracted dispersion dropping all the gluonic condensates, which should give solutions similar to the quenched lattice calculations [22], if our arguments are correct. No light glueball solution is found, as in the present lattice calculations. The solutions are indeed quite sensitive to the instanton parameters. A stable solution for a glueball with a mass of about 1560 MeV is shown in Fig. 4, with parameters instanton size = 1.28 GeV^{-1} and density = .00018 GeV^4. If one uses instanton parameters closer to or equal to the standard ones given above the glueball solutions are heavier, but we do not find consistency in the sense of the mass range of the Borel plateau of the sum rules including the mass of the solution. In other words with a pure gluonic instanton model we do not find stable solutions for a scalar glueball. This is consistent with our observations that in the 1-2 GeV regions the stable glueballs must have a scalar meson component. Note that recent lattice glueball solutions find the lightest scalar glueball mass at about 1700 MeV [23, 24].

We conclude that stable scalar glueball solutions in the range of 400-600 MeV are found if one includes the higher-dimensional gluonic condensates, and to the extent that the sum rule method is reliable there is a light glueball that could be part of the strongly coupled glueball/sigma system. Moreover, we find a plausable explanation of the present lattice calculations not finding a light glueball.

3.2 Mixed Scalar Mesons

For the solutions to the unsubtracted sum rules our most striking new result in the 1-2 GeV region is that there are now, with instantons, no stable solutions in which there is a dominant scalar meson ($q\bar{q}$) component with an even small admixture of a scalar glueball. This is in contrast to our earlier result [2] in which the only solutions obtained after the inclusion of the mixing given in Eq.(10) had a mixing of the order of 20 %. The process of the loop of $q\bar{q}$ in which the propagators are those of quarks in the background instanton-antiinstanton medium seems to replace the physical input of the glueball. This seems to be physically reasonable, but is not obvious. We obtain a second very stable solution with 80% glueball and 20% meson, which is in the region of the $f_o(1500)$; however, in contrast to the purely meson solution, the mainly glueball solution depends strongly on the choice of parameters. In fact, for the standard instanton liquid model there are no consistent solutions.

4 Discussion

For the subtracted sum rules, which are given by Eq.(8) with the addition of the instanton contributions to the glueball correlator, given by Eqs.(16,17), we find good solutions in the 400-600 MeV region as has been found in Refs.[6, 2, 7] without instantons. The instanton terms play a major role, and our solutions are more stable than those found earlier[2] and as expected less sensitive to the values of the higher-order gluonic condensates $\Gamma^{(6)}$ and $\Gamma^{(8)}$. We also have shown that if one only includes instantons without higher-dimensional gluon condensates there are no solutions for light glueballs. Moreover, we find no stable self-consistent scalar glueballs with the parameters of the instanton liquid model without scalar meson admixing. This suggests that when accurate unquenched lattice calculations are carried out that successfully account for the higher-dimension gluonic condensates they should also find a scalar glueball in the region of the sigma, about 500 MeV. Our results for a glueball in the 500 MeV region anddifficulty in obtaining consistent solutions in the 1-2 GeV region are consistent with the work of Ref. [7]. Recently, a calculation [20] using a double subtracted dispersion relation with both instantons and gluonic condensates find a scalar glueball solution in the 1.5 GeV region.

If our sigma/glueball conjecture is correct it makes the experimental study of branching ratios with channels containing sigmas for various glue-rich processes, such as hybrid decays and reactions dominated by Pomeron exchange, as well as heavy meson decays[25] most interesting.

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