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Approximate Equations of Diffusion and Magnetic Deflection in Radial-Drift TPC

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The objective is, for purposes of realistic estimates, to give simple expressions for the main novel effects in a radial-drift TPC: (1) The effect of the radial electric field $\vec{E}$ on the diffusion of the electron cloud. (2) The effect of the stray magnetic field $\vec{B}$ (from the central TPC) on the trajectory of the cloud as it drifts out radially.

1 Diffusion

The diffusion constant σ is proportional to $\sqrt{(\varepsilon_k/E)}$, where ε_k is the characteristic energy of the gas mixture. For many gas mixtures, ε_k increases with E. We approximate σ as independent of radius in order to get a simple analytic result. In an ordinary uniform $\vec{E}$, as the electron cloud drifts under $\vec{E}$, it also diffuses with an rms cloud size described by $\sigma = \sqrt{(< v^2 > \tau t)} = \sqrt{2\varepsilon_k x/eE)}$, where τ is the mean time between collisions, x is the distance drifted, and t is the time of drift. The $< v^2 >$ comes from $\varepsilon_k = m < v^2 > /2$. This is a random walk, superposed on the drift, with a Gaussian cloud characterized by σ. Use cylindrical coordinates $(r,\ z,\ \phi)$, with $\hat{z}$ along the beam direction, and the radial E's in the plane perpendicular to $\hat{z}$.

We divide the radial drift path into a large number of equal length segments. In traversing segment j, the already accumulated Gaussian shape has its width fanned out (increased) in the ϕ direction by the factor r_{j+1}/r_j. Added to that, in quadrature, is the effect of the diffusion in this segment. So the width (ϕ direction) of the Gaussian is

$$w_{j+1}^2 = w_j^2 \left(\frac{r_{j+1}}{r_j}\right)^2 + \sigma^2\delta \tag{1}$$

where the constant $\delta = (r_{j+1} - r_j)$. In justifying Eq. (1), recall that the convolution of a Gaussian of width α and a Gaussian of width β is a new Gaussian of width $\sqrt{(\alpha^2 + \beta^2)}$.

Start with a cloud of zero width, and iterate Eq. (1) many times, N, to represent successive stages of the drift. The result is

$$w_N^2 = \sigma^2 \delta \sum_{j=1}^{N} \left(\frac{r_N}{r_j} \right)^2 \tag{2}$$

The above sum can be replaced by the integral, in the limit $N >> 1$. The result for the width in the ϕ direction is

$$w = \sigma \sqrt{[(r_f - r_i) r_f / r_i]} \tag{3}$$

for a radial drift from radius r_i to r_f.

As a check, a program was run with a DO loop of N=1000 interactions of Eq. (1) in drifting from $r_i = 12$ cm to $r_f = 47$ cm, the approximate dimensions of the test TPC, under construction. The results agree with Eq. (3) to a fractional accuracy of 0.001.

Note, that Eq. (3) reduces to the expected result for a rectangular TPC in the limit $r_f/r_i >> 1$, while keeping $(r_f - r_i)$ fixed.

Expecially note how the spatial resolution of a radial-drift TPC is enhanced over that of a rectangular TPC. Consider a pair of tracks at the same r_i, but separated in the ϕ direction by some small distance. After drifting a distance $(r_f - r_i)$, the ratio (radial TPC to rectangular TPC) of the cloud separations (ϕ direction) is r_f/r_i, but the ratio of the sizes of the clouds is only $\sqrt{(r_f/r_i)}$. Thus the advantage for the radial-drift TPC in the ratio of cloud separation to cloud size increases as $\sqrt{(r_f/r_i)}$. This is most favorable for tracks with small r_i where the tracks are most dense. In the worst case, $r_f/r_i \to 1$, the radial drift and the rectangular TPC have the same resolution.

What is the effect of an initial cloud size? Or more generally, suppose that we have calculated the cloud size by Eq. (3) at some radius r_b, and want to change to a new value of σ for the next large (perhaps) interval of r. The cloud shape at r_b is a Gaussian with a known width w_b. There is some hypothetical radius r_0 from which a cloud of zero width could originate in order to form the size w_b at r_b. That r_0 is obtained from Eq. (3) as

$$r_0 = r_i \left[1 + \frac{(w_0/\sigma)^2}{r_i} \right]^{-1} \tag{4}$$

Hence a finite cloud size w_b can be propagated onward in radius by using Eq. (3) in which the effect of the finite w_b is included by replacing the starting radius r_i in Eq. (3) by the hypothetical r_0 of Eq. (4).

The algorithm, above, can easily be adapted to the case where σ varies with radius because of the r^{-1} electric field. Juan Romero (U.C. Davis) has made a very flexible program (to be reported) with the last term on the RH side of Eq. (1) replaced at each radial interval j

by a Monte Carlo representation of the diffusion in that interval. When his program is run with σ realistically varying with r, the results agree to 10% with those of Eqs. (3) and (4) with a single σ chosen by least squares fitting. When his program is run with constant σ, the agreement with Eqs. (3) and (4) is within his Monte Carlo statistics.

In the absence of magnetic field, for the usual gas mixtures the longitudinal and transverse diffusion constants are nearly equal. Thus for the approximation of σ being independent of radius, the diffusions in the z- and r- directions are also represented by Eq. (3), but with r_f/r_i set equal to unity, as for a rectangular TPC.

2 Effect of Magnetic Field

At the site of a drifting electron cloud in the external TPC, the stray magnetic field $\vec{B}$ from the central TPC will, to high accuracy, be in the plane containing the beam and the electric field vector experienced by that cloud. The Langevin equation for the drift velocity v_D of an electron is

$$\vec{v}_D = \frac{\mu E}{1+(\omega\tau)^2}\left[\hat{E} + (\hat{E}\times\hat{B})(\omega\tau) + \hat{B}(\hat{E}\cdot\hat{B})(\omega\tau)^2\right] \tag{5}$$

where $\omega = Be/m$ (cyclotron frequency of the electron), and a carat denotes a unit vector.

Since $\mu B = \omega\tau$, we can evaluate $\omega\tau$ from the drift velocity at a given E. The result is $\omega\tau << 1$ for any B likely to be experienced by the external TPC. Thus the third term of Eq. (5) is completely negligible. The second term produces drift at an angle $\phi_L \equiv \tan^{-1}(\omega\tau|\hat{E}\times\hat{B}|)$ measured in the $(\hat{E}\times\hat{B})$ (or ϕ) direction for $\vec{E}$. This is called the Lorentz angle when $|\hat{E}\times\hat{B}| = 1$.

In order to obtain a simple analytic result, assume that $\vec{B}$ and μ are uniform. In a rectangular TPC, $\vec{E}$ has a fixed direction, so the drift is in a straight line at the angle ϕ_L. In a radial-drift TPC, as the cloud drifts a short distance at the angle ϕ_L from $\vec{E}$ it reaches a new azimuth, with a new direction of $\vec{E}$. This results in a slightly spiral-like effect, as the direction of drift changes (in ϕ) with radius.

Figure 1 shows that

$$d\phi = \frac{(\phi_L\ dr)}{r} \tag{6}$$

during a radial drift through the infinitesimal radial interval dr. When integrated, this gives

$$\phi_f = \phi_L\ ln\left(\frac{r_f}{r_i}\right) \tag{7}$$

for a drift from radius r_i to r_f in a radial-drift TPC. Thus the cloud at radius r_f is displaced in the ϕ direction by the distance $r_f\phi_f$ from its location if B were zero. Figure 2 shows the effect.

To check this, a program with a 1000-pass DO loop was run in which at each pass the new r and ϕ were calculated according to Fig. 1. The fractional discrepancy between Eq. (7) and the program results is 0.001.

For a rectangular TPC, Eq. (7) is correct but not useful because of the cylindrical coordinates. However, in a rectangular TPC, the trajectory is simply linear at angle ϕ_L from E.

In the real case, the values of $\vec{B}$, $\omega\tau$ and $|\hat{E} \times \hat{B}|$ vary with radius, but this variation can easily be incorporated into the program. This has been done in the aforementioned program of Romero, and those results appear to agree with Eq. (7) when B is kept constant.

In the experimental environment at RHIC, this magnetic deflection can be incorporated into a lookup table of deflection (mm) vs drift time (i.e., r_i) in the same way that would be required for a rectangular TPC. For either type of TPC, ϕ_L will not be constant in the fringe field where the external TPC is located.

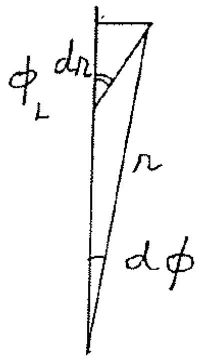

Fig. 1: Geometry for Eq (6).

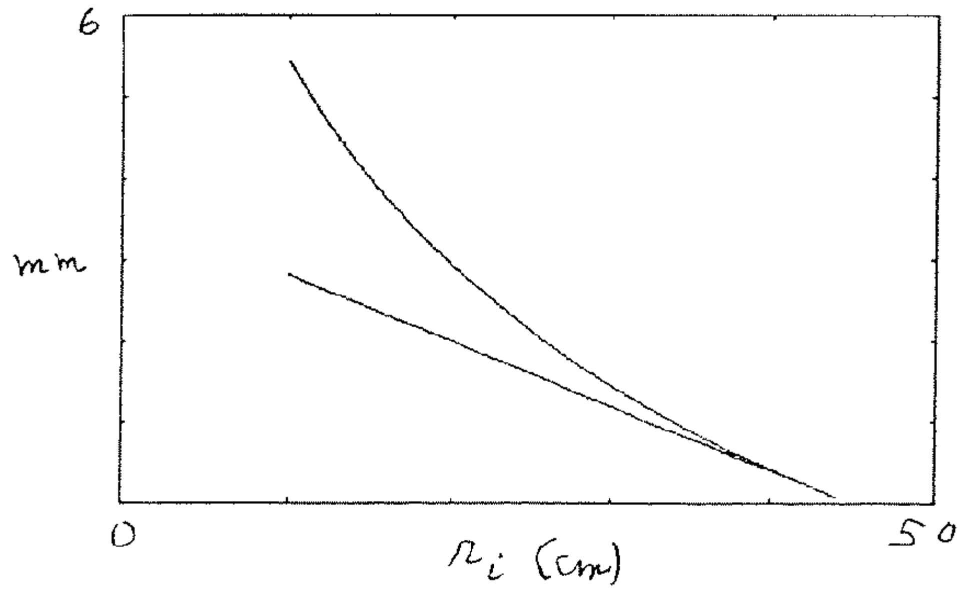

Fig. 2: Deflection (mm from r_f∗Eq (7)) at $r_f = 45$ cm for various r_i; $\phi_L = 0.01$ is approximate value for $b = 350$ Gauss with 80% Ar + 15% isobutane + 5% methylal. Straight line is corresponding deflection for rectangular TPC.