1 s-dependence of $\Gamma(s)$

The phase space for the 4π channel is strongly s-dependent and it is logically necessary for consistency to include this s-dependence into the Breit-Wigner denominator. We have calculated the s-dependence of the Lorentz invariant phase space $\rho(s)$ for $\sigma\sigma$ and $\rho\rho$ using equn. (40) of Ref. [1]:

$$\rho_{4\pi}(s) = \int_{4m_\pi^2}^{(\sqrt{s}-2m_\pi)^2} \frac{ds_1}{\pi} \int_{4m_\pi^2}^{(\sqrt{s}-\sqrt{s_1})^2} \frac{ds_2}{\pi} \frac{8|q|\ |q_1|\ |q_2|}{\sqrt{s\ s_1\ s_2}} |T_1(s_1)|^2 |T_2(s_2)|^2 \exp(-\alpha|q|^2). \quad (1)$$

Here q_1 and q_2 are momenta of pions from decay of each σ or ρ in its rest frame, and q is the momentum of the σ or ρ in the rest frame of $f_0(1370)$; T_1 and T_2 are amplitudes for ρ or σ decays. Fig. 1 shows $\rho(s)$ for a variety of form factors $\exp(-\alpha q^2)$, with α varying from zero to 6 GeV^{-2}. The exponential affects the phase space strongly only for masses far above that relevant to the present data, but it does affect the slope of the phase space as a function of $s = M^2$, even for low masses. The curves of Fig. 1 may be parametrised by the empirical form:

$$\rho(s) = \frac{A + Bs + Cs^2}{1 + \exp[D(E - s) + Fs^2 + Gs^3]}. \quad (2)$$

Table 1 gives values of the parameters.

The Breit-Wigner form we take for $f_0(1370)$ is:

$$f = \frac{1}{s - M_0^2 - m(s) + iM_0[\Gamma_{2\pi} + \Gamma_{KK}(s) + \Gamma_{\eta\eta}(s) + \Gamma_{\rho\rho}(s) + \Gamma_{\sigma\sigma}(s)]}. \quad (3)$$

We take $\Gamma_{2\pi}$ to be constant, but take all other Γ to be proportional to the available phase space $\rho(s)$ for decay, including the exponential form factor. The corresponding K-matrix elements are $K_{ij} = g_i g_j/(s - M_0^2)$, (where g are coupling constants), so M_0 is the mass of the K-matrix pole.

The quantity $m(s)$ in the denominator of equn. (3) is a dispersive correction to the mass:

$$m(s) = \frac{(s - M_0^2)}{\pi} \int ds' \frac{M_0 \Gamma_{total}(s')}{(s' - M_0^2)(s' - s)}. \quad (4)$$

This term makes equn. (3) fully analytic.

The contributions to $\Gamma(s)$ need input from other sources. We adopt coupling constants fitted to a large data-base by Anisovich et al. [2]. For this solution, coupling constants are such that, at the $f_0(1370)$ third-sheet pole, $\Gamma_{2\pi} : \Gamma_{KK} : \Gamma_{\eta\eta} : \Gamma_{4\pi} = 1 : 0.24 : 0.13 : 4.96$; the value of $\Gamma_{2\pi}$ is close to the maximum

Channel	α	A	B	C	D	E	F	G
$\sigma\sigma$	0	0.7535	-3.1275	3.1369	2.7698	1.6304	0.2798	-
	1	0.4981	-2.1283	2.2211	3.6277	1.4193	0.4251	-
	2	0.3547	-1.6050	1.7969	4.5431	1.2896	0.6618	-
	3	0.0791	-0.7415	1.3007	5.3429	1.2657	0.8117	-
	4	-0.2134	0.1223	0.8760	5.7038	1.2879	0.8638	-
	5	-0.3846	0.6540	0.5853	5.8868	1.3001	0.8786	-
	6	-0.4498	0.8857	0.4064	5.9783	1.2957	0.9027	-
$\rho\rho$	0	-3.2278	7.7632	0.1665	9.9221	1.1883	3.1276	-0.5343
	1	-4.3023	10.6713	-1.1290	9.0618	1.1981	3.1859	-0.5477
	2	-4.3479	11.1398	-1.6542	9.9198	1.1411	3.7098	-0.6547
	3	-3.9086	10.5712	-1.8097	11.3529	1.0632	4.5717	-0.8255
	4	-2.5078	9.1625	-1.7253	13.5015	0.9881	5.7774	-1.0523
	5	5.9635	3.6617	-0.9976	16.0788	0.9717	6.8497	-1.2092
	6	11.3862	-1.2680	-0.3091	16.4636	0.9916	6.8823	-1.1986

Table 1
Parameters in equn. (2) with s in GeV^2.

value consistent with absence of structure in Cern-Munich data around 1300–1370 MeV [1]. Magnitudes of widths in equn. (3) are adjusted to reproduce these values. The fit to the present data is actually quite insensitive to the coupling to $\pi\pi$, $\eta\eta$ and KK, but the following discussion comments on the fits to $f_0(1370)$ in other data.

The line shape predicted for the 2π and 4π channels is illustrated in Fig. 2 for the parameters of our final fit. The 2π and 4π cross sections are shown by full and dashed lines, normalised to 1 at each peak. Both peaks are somewhat asymmetric. The 2π channel is cut off strongly at higher masses by the rapidly increasing 4π width; the fall above the peak is faster than the rise to the peak. The converse applies to the 4π channel, because of the increasing 4π phase space. As a measure of the position and width of each peak, we determine the mass at half-height on each side of the peak; then the average mass $\bar{M}$ is taken as the mean of these values and the mean width $\bar{\Gamma}$ as the difference. These are approximately the values which will be obtained in fits with a Breit-Wigner amplitude of constant width. The 4π peak is centred 50–100 MeV above the 2π peak, because of the rapidly rising phase space for 4π.

The dash-dotted curve on Fig. 2 shows the line-shape of $f_0(1370)$ as it will appear in $\bar{p}p \to \eta(4\pi)$, modified by the available phase space; in this calculation, the cross section is multiplied by k/m, where k is the momentum with which $f_0(1370)$ is produced in $\bar{p}p \to f_0\pi$.

A somewhat surprising result is that it is not possible to find a reasonable set of parameters giving a full-width $\bar{\Gamma}$ greater than ~ 260 MeV; a smaller value appears more likely. Table 2 gives values of $\bar{M}$ and $\bar{\Gamma}$ for a variety of values of $\Gamma_{2\pi}$ and the parameter α appearing in the form factor. All are evaluated for a value of $M_0 = 1400$ MeV for the resonance mass. The narrow width is analogous to that observed for $a_0(980)$ and $f_0(980)$ and arises from damping of the resonance by the strong 4π channel, i.e. by the effect of unitarity.

The first surprise is that $\bar{\Gamma}$ for both 2π and 4π channels varies only slowly with the magnitude of $\Gamma_{2\pi}$. The second surprise is that the magnitude of α has a comparable effect. We now describe how these results arise.

α $(\mathrm{GeV})^{-2}$	$\Gamma_{2\pi}$ (MeV)	$\bar{M}_{2\pi}$ (MeV)	$\bar{\Gamma}_{2\pi}$ (MeV)	$\bar{M}_{4\pi}$ (MeV)	$\bar{\Gamma}_{4\pi}$ (MeV)
1	30	1349	159	1396	172
	60	1333	177	1391	188
	120	1322	189	1387	199
3	30	1332	182	1393	202
	60	1308	205	1383	220
	120	1290	222	1375	232
6	30	1318	200	1390	232
	60	1283	226	1374	251
	120	1259	242	1359	260

Table 2
Values of $\bar{M}$ and $\bar{\Gamma}$ for a variety of values of α and $\Gamma_{2\pi}$.

The dispersive correction $m(s)$ is evaluated with a subtraction on resonance. Any constant contribution to $\Gamma(s)$ makes no contribution to $m(s)$. For our final fit, $m(s)$ is shown in Fig. 3. It makes a large contribution to the Breit-Wigner denominator and its magnitude is controlled almost completely by $\Gamma_{4\pi}$, which varies most rapidly with s. The approximately linear variation with s of both $m(s)$ and $\Gamma_{4\pi}$ overwhelms the effect of the term $(s - M^2)$ in the Breit-Wigner denominator. In considering the mass range around 1300–1400

MeV, it is therefore helpful to rewrite equn. (3) as:

$$f \propto \frac{1/\Gamma_{total}(s)}{(s - M_0^2)/\Gamma_{total} - m(s)/\Gamma_{total} - iM_0}. \tag{5}$$

The effect of varying the strength of the coupling constants appears only in the first term in the denominator; when Γ_{total} is large, variations renormalise the overall strength of the Breit-Wigner amplitude, but have only a small effect on its shape. The width of each peak is instead sensitive to the slope of $\Gamma_{4\pi}$ against s through $m(s)$. Hence it is sensitive to the form factor and α; larger widths are obtained with slower variation of $\Gamma_{4\pi}$, hence a stronger form factor.

Several determinations of the resonance mass from 4π data favour values around 1390 MeV, while determinations from 2π data cluster around 1315 MeV with a width of ~ 230 MeV. This difference is in accord with the differences displayed in Table 2. The pole position is given by the mass at which the magnitude of the Breit-Wigner amplitude varies fastest. Since $\Gamma_{2\pi}$ is taken to be constant, the pole position is given by the fit to the 2π cross section, i.e. close to $1315 - i115$ MeV.

To select a set of values of $\Gamma_{2\pi}$ and α, we argue as follows. Most resonances have widths of typically 250 MeV, so if the 2π width is 20% of this, as in Ref. [2], a reasonable expectation is a value $\Gamma_{2\pi} \simeq 50$ MeV. Secondly, the form factor $\exp(-\alpha q^2)$ corresponds to a Gaussian source of radius R. With R in fm, $\alpha = (R/0.19732)^2/6$ GeV^{-2}. A radius of 0.5–0.8 fm corresponds to $\alpha = 1.1$ to 2.7 GeV^{-2}. We choose $M_0 = 1400$ MeV, $\Gamma_{2\pi} = 60$ MeV, and $\alpha = 3$ GeV^{-2} to obtain $\bar{M}_{2\pi} = 1308$ MeV, $\bar{\Gamma}_{2\pi} = 205$ MeV. This is sufficiently close to values obtained from 2π data. The fit to present data is not sensitive to variations of parameters over the range shown in Table 2. However, it is not possible to obtain a width as large as has been determined in many analyses of 4π data without unreasonably large values of $\Gamma_{2\pi}$ or α. It seems possible that these analyses have been affected by interferences between $f_0(1370)$ and $f_0(1500)$ or overlap of these two resonances.

References

[1] D.V. Bugg, A.V. Sarantsev and B.S. Zou, Nucl. Phys. B471 (1996) 59.

[2] V.V. Anisovich and A.V. Sarantsev, Phys. Lett. B382 (1996) 429; V.V. Anisovich et al., hep-ph/9711319, Phys. of Atomic Nuclei (in press).

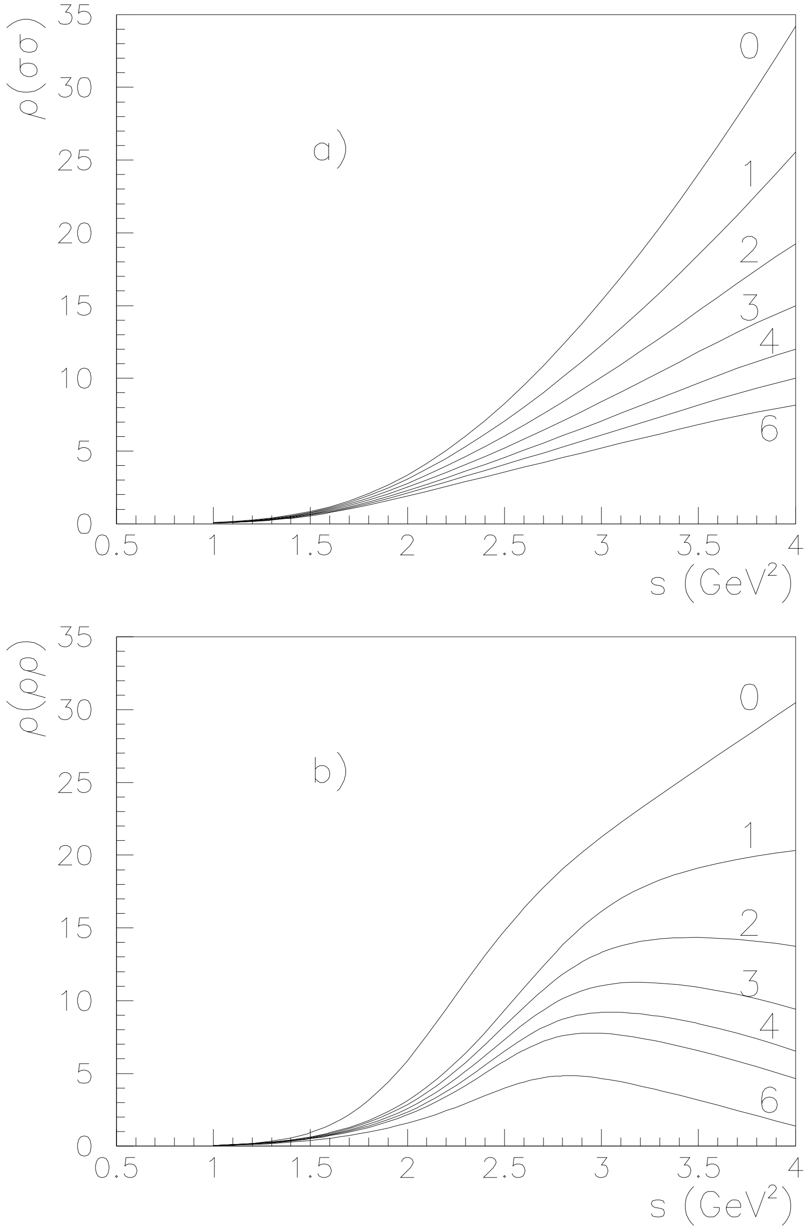

Fig. 1. The phase space for (a) $\sigma\sigma$, (b) $\rho\rho$ for values of α in the form factor from 0 to 6 GeV^{-2}.

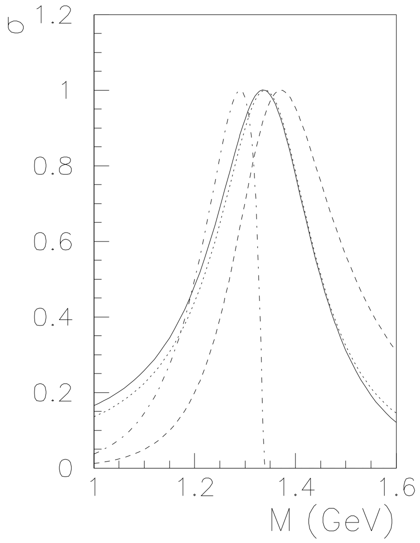

Fig. 2. The shape of $f_0(1370)$: full curve in the 2π channel, dashed curve in the 4π channel; dot-dashed curve as it appears in the 4π channel with the phase space available in $\bar{p}p \to \eta f_0$; the dotted curve shows a Breit-Wigner resonance of constant width centred at the same mass as the 2π curve.

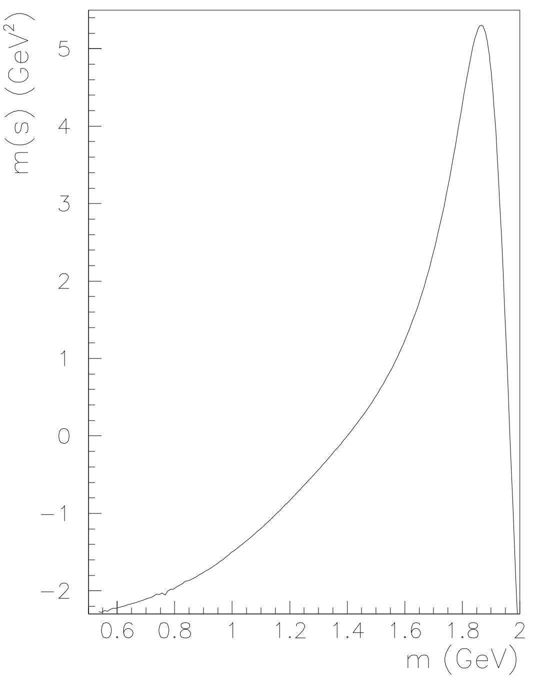

Fig. 3. The dispersive correction $m(s)$ to the mass.