$K \to \pi\pi$ and a light scalar meson

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We explore the $\Delta I = \frac{1}{2}$ rule and ϵ'/ϵ in $K \to \pi\pi$ transitions using a Dyson-Schwinger equation model. Exploiting the feature that QCD penguin operators direct K_S^0 transitions through 0^{++} intermediate states, we find an explanation of the enhancement of $K \to \pi\pi_{I=0}$ transitions in the contribution of a light σ-meson. This mechanism also affects ϵ'/ϵ.

I. INTRODUCTION

The $\Delta I = \frac{1}{2}$ rule is an empirical observation: the widths for nonleptonic decays of kaons and hyperons that change isospin by one-half unit are significantly larger than those for other K and Λ transitions; e.g., [1]

$$\Gamma_{K_S^0 \to (\pi\pi)}/\Gamma_{K^+ \to \pi^+\pi^0} = 660\,. \tag{1}$$

In terms of the amplitudes $M_{K_S^0 \to \pi^+\pi^-}$ and $M_{K_S^0 \to \pi^0\pi^0}$ that describe $K_S^0 \to \pi\pi$ transitions, the pure isospin-zero and isospin-two $\pi\,\pi$ final states are described by

$$A_0 = \tfrac{1}{\sqrt{6}}(2M_{K_S^0 \to \pi^+\pi^-} + M_{K_S^0 \to \pi^0\pi^0})\,, \tag{2}$$

$$A_2 = \tfrac{1}{\sqrt{3}}(M_{K_S^0 \to \pi^+\pi^-} - M_{K_S^0 \to \pi^0\pi^0})\,, \tag{3}$$

and the ratio in Eq. (1) corresponds to:

$$1/w := \mathrm{Re}(A_0)/\mathrm{Re}(A_2) \approx 22\,. \tag{4}$$

The analogous amplitude ratio for S-wave $\Lambda \to \pi N$ transitions is $|A_{1/2}/A_{3/2}| \approx 80$.

The processes involved are nonleptonic weak decays so one necessarily encounters QCD effects in their analysis and the operator product expansion (OPE) can therefore be employed to good effect. Using the OPE the amplitude for a given transition is expressed as the expectation value of an effective Hamiltonian: $A = \langle \mathcal{H}_{\rm eff}\rangle = \sum_i a_i(\mu)\,\langle Q_i(\mu)\rangle$, where μ is a renormalisation point. The coefficients: $a_i(\mu)$, are calculable in perturbation theory and describe short-distance effects. However, the expectation values of the local effective operators: $\langle Q_i(\mu)\rangle$, contain the effects of bound state structure; i.e., long-distance QCD effects, and must be calculated nonperturbatively.

The transitions of interest herein are mediated by nonleptonic strangeness changing ($\Delta S = 1$) effective operators. The simplest:

$$Q_1 = \bar{s}_i O_\mu^- u_j\ \bar{u}_j O_\mu^- d_i\,, \tag{5}$$

$$Q_2 = \bar{s}_i\, O_\mu^- u_i\ \bar{u}_j\, O_\mu^- d_j\,, \tag{6}$$

with $O_\mu^\pm = \gamma_\mu(1\pm\gamma_5)$ and colour indices: $i, j = 1, \ldots, N_c$, have the flavour structure of the standard weak four-fermion current-current interaction, and there are eight other terms representing the QCD and electroweak (ew) penguin operators.[1] At least some of these must have large expectation values if the $\Delta I = \frac{1}{2}$ rule is to be understood.

Another quantity that may be much influenced by the $\Delta S = 1$ effective interaction is the ratio ϵ'/ϵ. The indirect CP violating parameter:

$$\epsilon := A(K_L \to \pi\pi_{I=0})/A(K_S \to \pi\pi_{I=0}) \tag{7}$$

measures the admixture of CP-even state in K_L: for $\epsilon = 0$, CP $|K_{L/S}\rangle = \mp\,|K_{L/S}\rangle$; i.e., they are CP eigenstates. ϵ appears to be primarily determined by short-distance contributions from the weak nonleptonic $\Delta S = 2$ effective interaction [2]. In contrast, ϵ' measures the phase of the heavy-quark CKM matrix elements in the standard model and

$$\frac{\epsilon'}{\epsilon} = \frac{1}{\sqrt{2}\,|\epsilon|}\,\mathrm{Im}\left(\frac{A_2}{A_0}\right)\,, \tag{8}$$

with $|\epsilon| = 0.002280$, experimentally.[2] A nonzero value of ϵ'/ϵ entails direct transitions between CP-even and CP-odd eigenstates. ϵ' is sensitive to the same penguin operators that contribute to the $\Delta I = \frac{1}{2}$ rule, and hence is likely to receive significant long-distance contributions. The current generation of experiments [3] appears to be consistent and an average value of the ratio is [4]

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = (2.1 \pm 0.46) \times 10^{-3}\,. \tag{9}$$

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[1] Q_1 results from QCD corrections to the weak current-current vertex. The penguin operators are generated by QCD and ew corrections but their flavour structure is different.

[2] In Eq. (8) we follow contemporary practice and make explicit the $\pi\pi$-scattering phase shifts: $\delta_{0,2}$, in factoring out the phase $\Phi_{\epsilon'} = \frac{\pi}{2} + \delta_2 - \delta_0$. Then, using the experimental observations: $\delta_0 \approx 37°$, $\delta_2 \approx -7°$, $\Phi_\epsilon \approx \frac{\pi}{4}$, one has $\Phi_{\epsilon'} - \Phi_\epsilon \approx 0$ and the imaginary part in Eq. (8) relates only to an explicit CP violating phase.

A standard form of the $\Delta S = 1$ effective interaction at a renormalisation scale $\mu = 1\,\mathrm{GeV}$ is

$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \tilde{G}_F \sum_{i=1}^{10} c_i(\mu)\, Q_i(\mu)\,, \tag{10}$$

where $\tilde{G}_F = G_F\, V_{us}^*\, V_{ud}/\sqrt{2}$, $c_i(\mu) = z_i(\mu) + \tau\, y_i(\mu)$, $\tau = -(V_{ts}^* V_{td})/(V_{us}^* V_{ud})$, and $V_{ud},\ldots$, are the CKM matrix elements. (Direct CP violation is a measure of $\mathrm{Im}(\tau)$.) The coefficients: $c_i(\mu)$, at next-to-leading order are quoted in Ref. [2], as are the operators: Q_i. We reproduce the coefficients in the appendix, Eq. (A11), but not the operators and note only that $Q_{3,4,5,6}$ are the QCD penguin operators; e.g.,

$$Q_6 = \bar{s}_i O_\mu^- d_j \sum_{q=u,d,s} \bar{q}_j O_\mu^+ q_i\,, \tag{11}$$

and $Q_{7,8,9,10}$ are the ew penguin operators; e.g.,

$$Q_8 = \tfrac{3}{2}\, \bar{s}_i O_\mu^- d_j \sum_{q=u,d,s} e_q\, \bar{q}_j O_\mu^+ q_i\,, \tag{12}$$

where e_q is the quark's electric charge (in units of the positron charge). The expectation value of the operators in Eq. (10); i.e., the long-distance contributions, are the primary source of theoretical uncertainty in the estimation of w and ϵ'/ϵ, Eqs. (4) and (8).

Herein we calculate the expectation values of the operators in Eq. (10) using the Dyson-Schwinger equation (DSE) model of Ref. [5]. The DSE framework is reviewed in Ref. [6] with some of the phenomenological applications described in Refs. [7–11]. It treats mesons as bound states of a dressed-quark and -antiquark with Bethe-Salpeter amplitudes describing their internal structure, and has already been used to explore CP violation in hadrons [12]. We describe the calculation and its elements in Sect. II, and present and discuss our results in Sect. III. Section IV is a brief recapitulation.

II. OPERATOR EXPECTATION VALUES

A. Charged kaon decay

The impulse approximation to the meson-meson transitions mediated by $\mathcal{H}_{\rm eff}^{\Delta S=1}$ is straightforward to evaluate; e.g., in the absence of ew penguins only the operators $Q_{1,2}$ contribute to $K^+ \to \pi^+\pi^0$ transitions and

$$\langle \pi^+(p_1)\pi^0(p_2)|Q_1|K^+(p)\rangle = \tfrac{1}{\sqrt{2}} \sum_{i=1,2} N_c^i\, T_i(p_1,p_2)\,, \tag{13}$$

$$T_1(p_1,p_2) = i\sqrt{2}\,\mathrm{tr}Z_2 \int_{k_2}^{\Lambda} O_\mu^-\, \chi_\pi(k_2; -\tfrac{1}{2}p_2, -\tfrac{1}{2}p_2) \tag{14}$$
$$\times\, 2\,\mathrm{tr}Z_2 \int_{k_1}^{\Lambda} O_\mu^-\, \chi_K(k_1; p_2, p_1)\, \Gamma_\pi(k_1; -p_1)\, S_u(k_1)\,,$$

$$i\,T_2(p_1,p_2) = 2\sqrt{2}\mathrm{tr}Z_2^2 \int_{k_1}^{\Lambda}\int_{k_2}^{\Lambda} O_\mu^-\, \chi_\pi(k_2; -\tfrac{1}{2}p_2, -\tfrac{1}{2}p_2) \tag{15}$$
$$\times\, O_\mu^-\, \chi_K(k_1; p_2, p_1)\, \Gamma_\pi(k_1; -p_1)\, S_u(k_1)\,,$$

with the trace over Dirac indices only, and

$$\chi_\pi(k;\ell_1,\ell_2) = S_u(k+\ell_1)\, \Gamma_\pi(k;\ell_1+\ell_2)\, S_u(k-\ell_2)\,, \tag{16}$$
$$\chi_K(k;\ell_1,\ell_2) = S_s(k+\ell_1)\, \Gamma_K(k;\ell_1+\ell_2)\, S_u(k-\ell_2)\,. \tag{17}$$

Here we use a Euclidean formulation with $\{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu}$, $\gamma_\mu^\dagger = \gamma_\mu$ and $p\cdot q = \sum_{i=1}^4 p_i q_i$. $\int_k^\Lambda := \int^\Lambda d^4k/(2\pi)^4$ is a mnemonic representing a translationally invariant regularisation of the integral, with Λ the regularisation mass-scale that is removed ($\Lambda\to\infty$) as the final stage of any calculation, and $Z_2(\mu,\Lambda)$ is the quark wave function renormalisation constant. $S_{f=u,s}$ are the dressed-quark propagators (we assume isospin symmetry) and $\Gamma_{H=K,\pi}$ are the meson Bethe-Salpeter amplitudes, both of which we discuss in detail in Sect. II B.

Using the Fierz rearrangement property:

$$\mathrm{tr}\big[O_\mu^- G_1 O_\mu^- G_2\big] = -\mathrm{tr}\left[O_\mu^- G_1\right]\, \mathrm{tr}\big[O_\mu^- G_2\big]\,, \tag{18}$$

where $G_{1,2}$ are any Dirac matrices, it is clear that $T_1 \propto T_2$. Furthermore, the analysis for Q_2 is similar and the result is the same so that

$$\begin{aligned}&\langle \pi^+(p_1)\pi^0(p_2)|(c_1Q_1 + c_2Q_2)|K^+(p)\rangle \\ &= \frac{c_1+c_2}{\sqrt{2}}\, N_c(N_c+1)\, T_1(p_1,p_2)\,.\end{aligned} \tag{19}$$

This can be simplified using [13]

$$f_\pi p_\mu = -\sqrt{2}\, N_c \mathrm{tr}Z_2 \int_k^\Lambda O_\mu^-\, \chi_\pi(k; -\tfrac{1}{2}p, -\tfrac{1}{2}p) \tag{20}$$

and [14]

$$\begin{aligned}&-(p+p_1)_\mu\, f_+^{K^+}(p_2^2) - p_{2\mu}\, f_-^{K^+}(p_2^2) = \\ &2N_c\mathrm{tr}Z_2 \int_{k_1}^\Lambda i O_\mu^-\, \chi_K(k_1;p_2,p_1)\, \Gamma_\pi(k_1;-p_1)\, S_u(k_1)\,,\end{aligned} \tag{21}$$

where $f_\pm^{K^+}$ are the $K_{\ell 3}$ semileptonic transition form factors, so that

$$\begin{aligned}&\langle \pi^+(p_1)\pi^0(p_2)|\mathcal{H}_{\rm eff}^{\Delta S=1}|K^+(p)\rangle \\ &= \frac{N_c+1}{\sqrt{2}\,N_c}\, \tilde{G}_F\, (c_1+c_2)\, \mathcal{M}_1(p_1,p_2)\,,\end{aligned} \tag{22}$$

$$\begin{aligned}&\mathcal{M}_1(p_1,p_2) = \\ &\quad f_\pi \left[p_2\cdot(p+p_1) f_+^{K^+}(p_2^2) + p_2^2\, f_-^{K^-}(p_2^2)\right]\end{aligned} \tag{23}$$

$$\approx f_\pi\, (m_K^2 - m_\pi^2)\,, \tag{24}$$

where the last line follows from [14] $f_+^{K^+}(-m_\pi^2) \approx -1.0$ and $m_\pi^2 f_-^{K^-}(-m_\pi^2) \approx 0$.

We can compare our result with the contemporary phenomenological approach to $K\to\pi\pi$ decays, which employs a parametrisation of $\mathcal{M}_1$:

$$\mathcal{M}_1 = f_\pi\, (m_K^2 - m_\pi^2)\, B_1^{(3/2)}\,, \tag{25}$$

with the parameter $B_1^{(3/2)}$ fixed by fitting the experimental width. One historical means of estimating $\mathcal{M}_1$ is to employ the vacuum saturation *Ansatz*, which yields $B_1^{(3/2)} = 1$. It is thus clear from Eqs. (22) and (24) that our impulse approximation is equivalent to this *Ansatz*.[3] However, agreement with the experimental value of $\Gamma_{K^+\to\pi^+\pi^0}$ requires $B_1^{(3/2)} \approx \frac{1}{2}$, as can be seen using Eq. (A12). Thus, while the impulse approximation is reliable for estimating the order-of-magnitude, it appears that an accurate result requires additional contributions.[4] However, our primary goal is to identify a plausible mechanism for an enhancement of $\pi\pi_{I=0}$ transitions and this level of accuracy is sufficient for that purpose. Hence we proceed by adopting the contemporary artifice and use

$$\mathcal{M}_1(p_1,p_2) := f_\pi\,(m_K^2 - m_\pi^2)\,B_1^{(3/2)},\; B_1^{(3/2)} = \tfrac{1}{2}\,. \quad (26)$$

In doing this we bypass the calculation of $B_1^{(3/2)}$, which our elucidation of the impulse approximation has identified as a real challenge for models whose basis is kindred to ours, and also for other approaches.

B. Propagators and Bethe-Salpeter amplitudes

Although the matrix element discussed above was expressed in terms of dressed u- and s-quark propagators, and π- and K-meson Bethe-Salpeter amplitudes, we obtained a model independent result without introducing specific forms. That is an helpful but uncommon simplification only encountered before in the study of anomalous processes such as $\pi^0 \to \gamma\gamma$ [17].

In general, as reviewed in Ref. [6], these quantities can be obtained as solutions of the quark DSE and meson Bethe-Salpeter equation. However, the successful study of an extensive range of low- and high-energy light- and heavy-quark phenomena [5,7–12] has led to the development of efficacious algebraic parametrisations. These were employed in Ref. [5] and we also use them herein.

The dressed-quark propagator is

$$S_f(p) = -i\gamma\cdot p\,\sigma_V^f(p^2) + \sigma_S^f(p^2)\,, \quad (27)$$

$$= \left[i\gamma\cdot p\,A_f(p^2) + B_f(p^2)\right]^{-1}\,, \quad (28)$$

$$\bar\sigma_S^f(x) = 2\,\bar m_f\,\mathcal{F}(2(x+\bar m_f^2)) \quad (29)$$

$$+\mathcal{F}(b_1^f x)\,\mathcal{F}(b_3^f x)\,\left[b_0^f + b_2^f\mathcal{F}(\varepsilon x)\right]\,,$$

$$\bar\sigma_V^f(x) = \frac{1}{x+\bar m_f^2}\,\left[1 - \mathcal{F}(2(x+\bar m_f^2))\right]\,, \quad (30)$$

with $\mathcal{F}(y) = (1-\mathrm{e}^{-y})/y$, $x = p^2/\lambda^2$, $\bar m_f = m_f/\lambda$, $\bar\sigma_S^f(x) = \lambda\,\sigma_S^f(p^2)$ $\bar\sigma_V^f(x) = \lambda^2\,\sigma_V^f(p^2)$. The mass-scale, $\lambda = 0.566\,\mathrm{GeV}$, and parameter values:[5]

	$\bar m$	b_0	b_1	b_2	b_3
u	0.00948	0.131	2.94	0.733	0.185
s	0.210	0.105	3.18	0.858	0.185

(31)

were fixed in a least-squares fit to light- and heavy-meson observables [5], with these dimensionless u,s current-quark masses corresponding to

$$m_u^{1\,\mathrm{GeV}} = 5.4\,\mathrm{MeV},\; m_s^{1\,\mathrm{GeV}} = 119\,\mathrm{MeV}\,. \quad (32)$$

This algebraic parametrisation combines the effects of confinement and dynamical chiral symmetry breaking with free-particle behaviour at large spacelike p^2 [8].

The dominant component of the π- and K-meson Bethe-Salpeter amplitudes is primarily determined by the axial-vector Ward-Takahashi identity [13,24]:

$$\Gamma_H(k^2) = i\gamma_5\,\tfrac{\sqrt{2}}{f_H}\,B_H(k^2)\,,\; H = \pi, K\,, \quad (33)$$

where $B_H := B_u|^{\bar m_u\to 0}_{b_0^u\to b_0^P}$ and [5]

$$b_0^\pi = 0.204\,,\; b_0^K = 0.319\,; \quad (34)$$

i.e., B_H is the quark quark mass function obtained from Eqs. (27)-(30) with $\bar m_f = 0$ and b_0^f replaced by the values indicated. With these dressed-propagators and Bethe-Salpeter amplitudes one obtains (in GeV)

	f_π	m_π	f_K	m_K
Calc.	0.146	0.130	0.178	0.449
Obs. [1]	0.131	0.138	0.160	0.496

(35)

and $\langle\bar qq\rangle^{1\,\mathrm{GeV}} = (0.220\,\mathrm{GeV})^3$.

C. Neutral kaon Decay

We now turn to the transitions $K_S^0 \to \pi^+\pi^-$, $\pi^0\pi^0$. In comparison with $K^+ \to \pi^+\pi^0$ there is a significant qualitative difference: all effective operators contribute to these transitions and furthermore the QCD penguin operators: $Q_{5,6}$, and ew penguin operators: $Q_{7,8}$, can

[3] This is an exact algebraic constraint, which has been overlooked by other authors and consequently violated in fitting $\Gamma_{K^+\to\pi^+\pi^0}$; e.g., Ref. [16] and references therein.

[4] The impulse approximation, with dressed-propagators and vertices, has proven quantitatively reliable in many other applications; e.g., Refs. [5,7–11], and in some cases corrections have been calculated and shown to be small [14,15]. The new feature herein is that the calculation is not self-contained; i.e., we presently rely on external input: the c_i in Eq. (10).

[5] $\varepsilon = 10^{-4}$ in Eq. (29) acts only to decouple the large- and intermediate-p^2 domains. The study used Landau gauge because it is a fixed point of the QCD renormalisation group and $Z_2 \approx 1$, even nonperturbatively [13].

direct the transition through 0^{++} intermediate states. This can be important because; e.g., Ref. [18] reports evidence of a broad scalar resonance in $\tau \to \nu_\tau \pi^- \pi^0 \pi^0$ decays:

$$m_{0^{++}} \approx 1.12\, m_K\,,\ \Gamma_{0^{++}\to\pi\pi} \approx 0.54\,\mathrm{GeV}\,, \tag{36}$$

and with $m_{0^{++}} \approx m_K$ such a resonance could provide a significant contribution to the nonleptonic K^0 decays. We explore this hypothesis by allowing such a contribution in our analysis: $K^0_S \to \sigma_{0^{++}} \to \pi\pi$.

Before proceeding further we note that a light scalar meson is a feature of DSE studies using a well-constrained rainbow-ladder truncation [19–21]. However, in the 0^{++} channel this truncation is very likely unreliable [22], a fact we expect is entangled with the phenomenological difficulties encountered in understanding the composition of scalar resonances below 1.4 GeV [23]. While the lack of a reliable DSE truncation for scalar mesons prevents an accurate calculation of their Bethe-Salpeter amplitude and mass, they are nevertheless describable by such amplitudes, which herein we parametrise as

$$\Gamma_\sigma(k;p) = I_D\, \tfrac{1}{\mathcal{N}_\sigma}\, \frac{1}{1+(k^2/\omega_\sigma^2)^2}\,, \tag{37}$$

where $I_D = \gamma_4^2$ and ω_σ is a width parameter. Γ_σ is normalised canonically and consistent with the impulse approximation ($q_\pm = q \pm \frac{1}{2}p$)

$$\begin{aligned} p_\mu = N_c\,\mathrm{tr}\int_q^\Lambda \Big[& \Gamma_\sigma(q;-p)\frac{\partial S(q_+)}{\partial p_\mu}\Gamma_\sigma(q;p)S(q_-) \\ & + \Gamma_\sigma(q;-p)S(q_+)\Gamma_\sigma(q;p)\frac{\partial S(q_-)}{\partial p_\mu}\Big]\Big|_{p^2=-m_\sigma^2}\,. \end{aligned} \tag{38}$$

We separate the Q_6 contribution to the $K^0_S \to \pi\pi$ transition into two parts and consider first the new class of contributions, which introduce the σ intermediate state:

$$\begin{aligned} & \langle \pi(p_1)\pi(p_2)|Q_6|K^0(p)\rangle = \\ & \langle \pi(p_1)\pi(p_2)|\sigma(p)\rangle\, D_\sigma(p^2)\, \langle \sigma(p)|Q_6|K^0(p)\rangle\,, \end{aligned} \tag{39}$$

where we represent σ propagation by

$$D_\sigma(p^2) = 1/[p^2 + m_\sigma^2] \tag{40}$$

and employ the impulse approximation for the $\sigma\pi\pi$ coupling

$$\begin{aligned} & M_{\sigma\pi\pi}(p_1,p_2) := \langle \pi(p_1)\pi(p_2)|\sigma(p)\rangle = \\ & 2N_c \mathrm{tr}\int_k^\Lambda \Gamma_\sigma(k;p)\, S_u(k_{++}) \\ & \times i\Gamma_\pi(k_{0+};-p_1)\, S_u(k_{+-})\, i\Gamma_\pi(k_{-0};-p_2)\, S_u(k_{--})\,, \end{aligned} \tag{41}$$

$k_{\alpha\beta} = k + (\alpha/2)p_1 + (\beta/2)p_2$, which provides the basis for the calculation of $\Gamma_{0^{++}\to\pi\pi}$. This combination of simple-pole propagator plus impulse approximation coupling to the dominant decay channel is phenomenologically efficacious; e.g., Refs. [10].

In impulse approximation

$$\begin{aligned} & \langle \sigma(p)|Q_6|K^0(p)\rangle = \sqrt{2}\, N_c^2 \times \\ & \mathrm{tr}\, Z_4^2 \int_{k_1}^\Lambda\int_{k_2}^\Lambda i\chi_K(k_1;\tfrac{1}{2}p,\tfrac{1}{2}p)\, O_\mu^+\, \chi_\sigma(k_2;-\tfrac{1}{2}p,-\tfrac{1}{2}p)\, O_\mu^-\,, \end{aligned} \tag{42}$$

with $Z_4(\mu,\Lambda)$ the mass renormalisation constant and $\chi_\sigma(k;\ell_1,\ell_2)$ an obvious analogue of $\chi_\pi(k;\ell_1,\ell_2)$ in Eq. (16). Using

$$\mathrm{tr}[G_1 O_\mu^+ G_2 O_\mu^-] = 2\,\mathrm{tr}[G_1(1-\gamma_5)]\,\mathrm{tr}[G_2(1+\gamma_5)] \tag{43}$$

this yields

$$\begin{aligned} & -\tfrac{1}{\sqrt{2}}\langle \sigma(p)|Q_6|K^0(p)\rangle = \\ & \left(\sqrt{2}\, N_c\, \mathrm{tr} Z_4 \int_{k_1}^\Lambda i\gamma_5\, \chi_K(k_1;\tfrac{1}{2}p,\tfrac{1}{2}p)\right) \\ & \times \left(\sqrt{2}\, N_c\, \mathrm{tr} Z_4 \int_{k_2}^\Lambda \chi_\sigma(k_2;-\tfrac{1}{2}p,-\tfrac{1}{2}p)\right)\,. \end{aligned} \tag{44}$$

From Refs. [13,24] we identify the first parenthesised term as the residue of the kaon pole in the pseudoscalar vertex:

$$\begin{aligned} & ir_K := \\ & \sqrt{2}\, N_c\, \mathrm{tr} Z_4 \int_{k_1}^\Lambda \gamma_5\ \chi_K(k_1;\tfrac{1}{2}p,\tfrac{1}{2}p) = \frac{f_K\, m_K^2}{m_u + m_s}\,. \end{aligned} \tag{45}$$

The second term is the scalar meson analogue in the scalar vertex but the vector Ward-Takahashi identity, which is relevant in this case, does not make possible an algebraic simplification. The integral and its μ-dependence must therefore be calculated. That is straightforward when the renormalisation-group-improved rainbow-ladder truncation is accurate; e.g., Refs. [11,13,19], but not yet for scalar mesons. This is where the simple *Ansatz* of Eq. (37) is useful: it yields a finite integral and we therefore suppress Z_4 to obtain

$$\begin{aligned} & \tfrac{1}{\sqrt{2}}\langle \sigma(p)|Q_6|K^0(p)\rangle \\ & = r_K\, \sqrt{2}\, N_c\, \mathrm{tr}\int_{k_2}^\Lambda \chi_\sigma(k_2;-\tfrac{1}{2}p,-\tfrac{1}{2}p) =: r_K\, r_\sigma(p^2)\,. \end{aligned} \tag{46}$$

The result for Q_5 is similar, but suppressed by a factor of $1/N_c$, and the contribution of the ew penguins: $Q_{7,8}$, can be obtained similarly.

The other class of contributions, which do not involve a 0^{++} intermediate state, can be evaluated following the explicit example of Q_1 presented above. Only two additional three-point functions arise:

$$\mathcal{G}_\pi^S(p_1,p_2) = \langle \pi(p_1)\pi(p_2)|(\bar{u}u + \bar{d}d)|0\rangle\,, \tag{47}$$

$$-\mathcal{G}_{K\pi}^S(p_1,p_2) = \langle \pi^-(p_1)|\,\bar{s}u\,|K^0(p)\rangle\,. \tag{48}$$

They are the scalar pion form factor and the scalar $K\pi$ transition form factor, respectively, and $\mathcal{G}_{K\pi}^S(p_1,p_2)$ can

be expressed without additional calculation in terms of the $K_{\ell 3}$ form factors [14]:

$$\mathcal{G}^S_{K\pi}(p_1,p_2) = \frac{p_1^2-p_2^2}{m_s-m_d}\left[f_+^K(-p_2^2)+\frac{p_2^2}{p_2^2-p_1^2}f_-^K(-p_2^2)\right], \quad (49)$$

a result which follows from the vector Ward-Takahashi identity. A preliminary result is available [25] for $\mathcal{G}^S_\pi(p_1,p_2)$, which takes the form anticipated from current algebra. That is to be expected because correctly truncated DSE models provide a good description of chiral symmetry and its dynamical breakdown, as illustrated in a study of $\pi\pi$ scattering [26]. This makes a calculation of $\mathcal{G}^S_\pi(p_1,p_2)$ unnecessary for our present study because we can adopt the form [27] [$(r^S_\pi) = 3.76\,\mathrm{GeV}^{-1}$]:

$$\mathcal{G}^S_\pi(p_1,p_2) = -4\frac{\langle\bar{q}q\rangle}{f_\pi^2}\left[1-\tfrac{1}{6}(r^S_\pi)^2(p_1+p_2)^2\right]. \quad (50)$$

The matrix elements for the $K\to\pi\pi$ transitions can all be written

$$M_{K\to\pi\pi} = M^{\rm qcd}_{K\to\pi\pi} + \alpha_{\rm em}\,M^{\rm ew}_{K\to\pi\pi}, \quad (51)$$

with the explicit forms given in the appendix and the pure isospin amplitudes defined in Eqs. (2), (3).

III. RESULTS AND DISCUSSION

Everything required for our calculation of the widths is now specified. There are two parameters: ω_σ in Eq. (37); and m_σ in Eq (40). We determine them in a least-squares fit to: $\Gamma_{K^0_S\to\pi^+\pi^-}$, $\Gamma_{K^0_S\to\pi^0\pi^0}$, taken from Ref. [1]; and $\Gamma_{\sigma\to(\pi\pi)}$ in Eq. (36), and obtain (in GeV)

	Obs.	Calc.
m_σ	$1.12\,m_K$	$1.14\,m_K$
ω_σ		0.611
$\Gamma_{\sigma\to(\pi\pi)}$	0.54	0.54
$\Gamma^{\times 10^{-15}}_{K^0_S\to\pi^+\pi^-}$	5.055 ± 0.025	5.16
$\Gamma^{\times 10^{-15}}_{K^0_S\to\pi^0\pi^0}$	2.305 ± 0.023	2.11
$\Gamma^{\times 10^{-15}}_{K^+\to\pi^+\pi^0}$	0.0112 ± 0.0001	0.0116

(52)

which is a relative error on fitted quantities of $< 4\%$.[6] This value of ω_σ corresponds to an *intrinsic* σ-meson size: $r^I_\sigma := 1/\omega_\sigma$, which is $0.84\,r^I_\rho$; i.e, 84% of that of the ρ-meson determined in Ref. [5].

[6] We used $G_F = 1.166\times10^{-5}\,\mathrm{GeV}^{-2}$, $V_{ts} = 0.0385$, $V_{td} = 0.0085$, $V_{us} = 0.220$, $V_{ud} = 0.975$, $\mathrm{Im}(V^*_{ts}V_{td}) = 0.000133$, and c_i obtained from Eq. (A11). With $\Gamma_\sigma(k;p)\propto\exp(-k^2/\omega_\sigma^2)$ instead of Eq. (37), $m_\sigma = 1.12\,m_K$, $\omega_\sigma = 0.694\,\mathrm{GeV}$ yields exactly the same results for the calculated quantities.

The widths in Eq. (52) are obtained from the calculated amplitudes (in GeV with m_K from Eq. (35))

$$|M_{K^0\to\pi^+\pi^-}| = 2.7\times10^{-7} = 5.9\times10^{-7}\,m_K, \quad (53)$$
$$|M_{K^0\to\pi^0\pi^0}| = 2.4\times10^{-7} = 5.4\times10^{-7}\,m_K, \quad (54)$$
$$|M_{K^+\to\pi^+\pi^0}| = 1.8\times10^{-8} = 4.0\times10^{-8}\,m_K. \quad (55)$$

For the pure isospin amplitudes we find (in GeV):

$$\mathrm{Re}(A_0) = 31.7\times10^{-8},\ \mathrm{Re}(A_2) = 1.47\times10^{-8}, \quad (56)$$

which are consistent with recent lattice estimates [28] and yield

$$1/w = 21.6. \quad (57)$$

Our analysis also yields values of the parameters: $B_i^{(1/2),(3/2)}$, used in phenomenological analyses to express the operator expectation values [2]. Of course, $B_1^{(3/2)} = 0.5$, as discussed in connection with Eq. (26) and, using the formulae in the appendix, we obtain algebraically: $B_1^{(1/2)} = B_2^{(1/2)} = B_3^{(1/2)} = B_4^{(1/2)} = B_2^{(3/2)} = B_1^{(3/2)}$. We also calculate

$$B_5^{(1/2)} = B_6^{(1/2)} = 1.43 + (17.9)_\sigma, \quad (58)$$

where the second term is the contribution of the σ-meson. The non-σ contribution is large because of the strength of the $K\pi$ transition form factor. If the vacuum saturation *Ansatz* is used to estimate the operator expectation values they are all $\equiv 1$. That method does not admit a σ-meson contribution.

Eliminating the ew penguin contributions yields a $< 1\%$ reduction in $1/w$, which is consistent with the the magnitude of $\alpha_{\rm em}$. Suppressing instead the σ-meson contribution, while not affecting $\Gamma_{K^+\to\pi^+\pi^0}$ of course (see Eq. (A1)), yields $\Gamma_{K^0_S\to\pi^+\pi^-} = 1.3\times10^{-16}\,\mathrm{GeV}$, $\Gamma_{K^0_S\to\pi^0\pi^0} = 1.1\times10^{-17}\,\mathrm{GeV}$, and $1/w = 2.9$.

The value of ϵ'/ϵ follows from Eq. (8). Suppressing the σ-meson and ew penguin contributions we obtain $\epsilon'/\epsilon = 128\times10^{-3}$, which is ~ 60-times larger than the experimental average in Eq. (9). Including the σ-meson we find 31.3×10^{-3}. To understand these results we note that Eq. (8) can be written

$$\frac{\epsilon'}{\epsilon} = -\tfrac{1}{\sqrt{2}}\frac{w}{|\epsilon|}\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\left\{1-\frac{1}{w}\frac{\mathrm{Im}A_2}{\mathrm{Im}A_0}\right\}, \quad (59)$$

which makes clear that the ratio is determined by $\mathrm{Im}(A_0)/\mathrm{Re}(A_0)$ *unless* $\mathrm{Im}(A_2)\neq 0$. Noting that $c_{1,2}$ are real, Eq. (A11), then it follows from Eq. (A1) that $\mathrm{Im}(A_2) = 0$ in the absence of ew effects. Hence our calculated results are large because the pre-factor in Eq. (59) is large. The dependence on the σ contribution is easily understood. The pre-factor is $\propto \mathrm{Im}(A_0)/\mathrm{Re}(A_0)^2$, which is large in the absence of the σ contribution even though $\mathrm{Im}(A_0)$ and $\mathrm{Re}(A_0)$ are individually small. The σ contribution adds simultaneously to $\mathrm{Im}(A_0)$ and $\mathrm{Re}(A_0)$ with

a magnitude $\sim$ 100-times larger than the original values. Hence the final ratio is sensitive only to the relative strength of the σ contributions, which is determined by the coefficients $c_{5,6}$.

Including both the σ and ew penguin contributions we obtain

$$\epsilon'/\epsilon = 31.7 \times 10^{-3}\,, \tag{60}$$

from which it is clear that the ew penguins are a correction of order α_{em} as one would naively expect. In this case $\mathrm{Im}(A_2) \neq 0$. However, as observed above, the σ-meson enhancement responsible for the $\Delta I = \frac{1}{2}$ rule affects the real and imaginary parts of A_0 simultaneously so that $(1/w)\,\mathrm{Im}A_2/\mathrm{Im}A_0$ remains negligible.

If we employ the artifice of an *ad hoc* suppression of the σ contribution to $\mathrm{Im}(A_0)$ while retaining it in $\mathrm{Re}(A_0)$; i.e., make the replacement

$$c_i\,\mathcal{M}_3 \to \mathrm{Re}(c_i\,\mathcal{M}_3),\ i = 5,6,7,8 \tag{61}$$

in Eqs. (A1)-(A6), we find

$$\epsilon'/\epsilon = 2.7 \times 10^{-3}\,. \tag{62}$$

This artifice is implicit in the phenomenological analyses reviewed in Ref. [2] and that is why Eq. (62) reproduces their order-of-magnitude. The small value is only possible because in this case $\mathrm{Im}(A_0)$ is not σ-enhanced and is therefore of the same magnitude as $\mathrm{Im}A_2/w \propto \alpha_{\rm em}/w$, due to the $1/w$ enhancement factor. That factor survives because $\mathrm{Re}(A_0)$ is still magnified as required in order to satisfy the $\Delta I = \frac{1}{2}$ rule. Currently we cannot justify this procedure.[7]

IV. EPILOGUE

We have demonstrated that estimating the $K \to \pi\pi_{I=2}$ matrix element using the impulse approximation is algebraically equivalent to using the vacuum saturation *Ansatz* and yields a result that is $\sim$ 2-times too large. The identification of a compensating mechanism that can correct for this overestimate is a contemporary challenge.

We have also shown that the contribution of a light scalar meson mediated by the QCD penguin operators: $Q_{5,6}$, is a plausible candidate for the long-range mechanism underlying the enhancement of $K \to \pi\pi_{I=0}$ transitions.[8] A good description of that enhancement *requires* a mass and width for this 0^{++} resonance that agree with those recently inferred [18], and the analysis is not sensitive to details of the model Bethe-Salpeter amplitude. However, this same mechanism yields a value of ϵ'/ϵ that is $\sim$ 15-times larger than the average of contemporary experimental results unless a means is found to suppress its contribution to $\mathrm{Im}(A_0)$.

If a light scalar resonance exists it will contribute in the manner we have elucidated and should be incorporated in any treatment of $K \to \pi\pi$.

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APPENDIX: COLLECTED FORMULAE

The matrix elements for the $K \to \pi\pi$ transitions are all of the form in Eq. (51) with

$$M^{\rm qcd}_{K^+\to\pi^+\pi^0} = \tfrac{1}{\sqrt{2}}\,\tilde{G}_F\,(1+\tfrac{1}{N_c})\,(c_1+c_2)\,\mathcal{M}_1\,, \tag{A1}$$

$$M^{\rm qcd}_{K^0_S\to\pi^+\pi^-} = \tilde{G}_F\,\Big\{[c_2+c_4+\tfrac{1}{N_c}(c_1+c_3)]\mathcal{M}_1 + 2\,(\tfrac{1}{N_c}c_5+c_6)\,(\mathcal{M}_2+\tfrac{1}{\sqrt{2}}\mathcal{M}_3)\Big\}\,, \tag{A2}$$

$$M^{\rm qcd}_{K^0_S\to\pi^0\pi^0} = \tilde{G}_F\,\Big\{[c_4-c_1-\tfrac{1}{N_c}(c_2-c_3)]\mathcal{M}_1 + 2\,(\tfrac{1}{N_c}c_5+c_6)\,(\mathcal{M}_2+\tfrac{1}{\sqrt{2}}\mathcal{M}_3)\Big\}\,, \tag{A3}$$

$$M^{\rm ew}_{K^+\to\pi^+\pi^0} = -\tfrac{1}{\sqrt{2}}\tilde{G}_F\,\Big\{\tfrac{3}{2}\Big[c_7+\tfrac{1}{N_c}c_8 - (1+\tfrac{1}{N_c})(c_9+c_{10})\Big]\,\mathcal{M}_1 + 3(\tfrac{1}{N_c}c_7+c_8)\,\mathcal{M}_2^b\Big\}\,, \tag{A4}$$

$$M^{\rm ew}_{K^0_S\to\pi^+\pi^-} = \tilde{G}_F\,\Big\{(\tfrac{1}{N_c}c_9+c_{10})\mathcal{M}_1 - (\tfrac{1}{N_c}c_7+c_8)\,(\mathcal{M}_2^a+2\mathcal{M}_2^b+\tfrac{1}{\sqrt{2}}\mathcal{M}_3)\Big\}\,, \tag{A5}$$

$$M^{\rm ew}_{K^0_S\to\pi^0\pi^0} = \tilde{G}_F\,\Big\{\Big[c_7+\tfrac{1}{N_c}c_8-(1+\tfrac{1}{2N_c})c_9 - \tfrac{1}{2}(1+\tfrac{2}{N_c})c_{10}\Big]\,\mathcal{M}_1 - (\tfrac{1}{N_c}c_7+c_8)(\mathcal{M}_2+\tfrac{1}{\sqrt{2}}\mathcal{M}_3)\Big\}\,, \tag{A6}$$

[7] NB. If this procedure is followed then $m_u \neq m_d$ isospin symmetry breaking effects also contribute significantly to ϵ'/ϵ.

[8] $Q_{5,6}$ mediated scalar diquark transitions: $(us)^{I=1/2}_{0+} \to (ud)^{I=0}_{0+}$, are the $s \to t$-channel interchange of the interaction that herein produces the σ-meson. They are a viable candidate for the mechanism that produces the $\Delta I = \frac{1}{2}$ enhancement for baryons. This was explored in Ref. [29], however, the requirement therein that diquarks also explain the enhancement for mesons appears unnecessarily cumbersome.

$$\mathcal{M}_2^a = r_K\, \mathcal{G}_\pi^S(p_1,p_2)\,,\ \mathcal{M}_2^b = r_\pi\, \mathcal{G}_{K\pi}^S(p_1,p_2)\,, \tag{A7}$$

$$\mathcal{M}_2 = \mathcal{M}_2^a - \mathcal{M}_2^b\,, \tag{A8}$$

$$\mathcal{M}_3 = r_K\, r_\sigma(p^2)\, D_\sigma(p^2)\, M_{\sigma\pi\pi}(p_1,p_2)\,, \tag{A9}$$

with $p^2 = (p_1+p_2)^2 = -m_K^2$, $p_1^2 = p_2^2 = -m_\pi^2$. These formulae make clear the operators that would be suppressed if N_c were large. Note that

$$M^{\rm qcd}_{K^+\to\pi^+\pi^0} = \tfrac{1}{\sqrt{2}}(M^{\rm qcd}_{K^0_S\to\pi^+\pi^-} - M^{\rm qcd}_{K^0_S\to\pi^0\pi^0})\,. \tag{A10}$$

This is not true of the complete amplitude.

In our calculations we use values of the coefficients that correspond to our choice of $\Lambda_{\rm QCD} \sim 0.2\,$GeV: $c_i = z_i + \tau\, y_i$, $\tau = -(V_{ts}^* V_{td})/(V_{us}^* V_{ud})$ with [2]

	z_i	y_i
1	−0.407	0.0
2	1.204	0.0
3	0.007	0.023
4	−0.022	−0.046
5	0.006	0.004
6	−0.022	−0.076
7	0.003	−0.033
8	0.008	0.121
9	0.007	−1.479
10	−0.005	0.540

(A11)

Using the alternative set listed in Ref. [2] then, with $m_\sigma = 1.06\, m_K$ and $\omega_\sigma = 0.670\,$GeV, we obtain results that differ from those in Eq. (52) by $\lesssim 1\%$, and $\epsilon'/\epsilon = 69.0 \times 10^{-3}$ primarily because y_5 in the alternative set is 2.6-times as large.

From the complete matrix elements: Eqs. (51) and Eqs. (A1)-(A6), we obtain the widths:

$$\Gamma_{K^+\to\pi^+\pi^0} = \mathcal{C}(m_K)\,|M_{K^+\to\pi^+\pi^0}|^2\,, \tag{A12}$$

$$\Gamma_{K^0_S\to\pi^+\pi^-} = 2\,\mathcal{C}(m_K)\,|M_{K^0_S\to\pi^+\pi^-}|^2\,, \tag{A13}$$

$$\Gamma_{K^0_S\to\pi^0\pi^0} = \mathcal{C}(m_K)\,|M_{K^0_S\to\pi^0\pi^0}|^2\,, \tag{A14}$$

$$\mathcal{C}(x) = \frac{1}{16\pi\, x}\sqrt{1-\tfrac{4m_\pi^2}{x^2}}\,, \tag{A15}$$

while the matrix element of Eq. (41) features in

$$\Gamma_{\sigma\to(\pi\pi)} = \tfrac{3}{2}\,\mathcal{C}(m_\sigma)\,|M_{\sigma\pi\pi}(m_\sigma^2; m_\pi^2, m_\pi^2)|^2\,. \tag{A16}$$

[1] Particle Data Group (C. Caso *et al.*), Eur. Phys. J. C **3**, 1 (1998).

[2] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[3] G.D. Barr *et al.* [NA31 Collaboration], Phys. Lett. **B 317**, 233 (1993); L.K. Gibbons *et al.* [E731 Collaboration], Phys. Rev. **D 55**, 6625 (1997); A. Alavi-Harati *et al.* [KTeV Collaboration], Phys. Rev. Lett. **83**, 22 (1999); P. Debu, seminar at CERN, 18/June/99, http://www.cern.ch/NA48).

[4] M. Fabbrichesi, "Estimating ϵ'/ϵ. A user's manual," hep-ph/9909224.

[5] M.A. Ivanov, Y.L. Kalinovsky and C.D. Roberts, Phys. Rev. **D 60**, 034018 (1999).

[6] C.D. Roberts and A.G. Williams, Prog. Part. Nucl. Phys. **33**, 477 (1994).

[7] P.C. Tandy, Prog. Part. Nucl. Phys. **39**, 117 (1997);

[8] C.D. Roberts, Fiz. Élem. Chastits At. Yadra **30**, 537 (1999) (Phys. Part. Nucl. **30**, 223 (1999)).

[9] G. Hellstern, R. Alkofer, M. Oettel and H. Reinhardt, Nucl. Phys. **A 627**, 679 (1997); J.C. Bloch, C.D. Roberts, S.M. Schmidt, A. Bender and M.R. Frank, "Nucleon form factors and a nonpointlike diquark," nucl-th/9907120, to appear in Phys. Rev. C (Rapid Comm.).

[10] L.C.L. Hollenberg, C.D. Roberts and B.H.J. McKellar, Phys. Rev. **C 46**, 2057 (1992); K.L. Mitchell and P.C. Tandy, Phys. Rev. **C 55**, 1477 (1997); M.A. Pichowsky, S. Walawalkar and S. Capstick, Phys. Rev. **D 60**, 054030 (1999).

[11] P. Maris and P.C. Tandy, "Bethe-Salpeter study of vector meson masses and decay constants," nucl-th/9905056, to appear in Phys. Rev. C.

[12] M.B. Hecht and B.H. McKellar, Phys. Rev. **C 57**, 2638 (1998); "Off mass shell effects in hadron electric dipole moments," hep-ph/9906246, to appear in Phys. Rev. C.

[13] P. Maris and C.D. Roberts, Phys. Rev. **C 56**, 3369 (1997).

[14] Yu.L. Kalinovsky, K.L. Mitchell and C.D. Roberts, Phys. Lett. **B 399**, 22 (1997).

[15] R. Alkofer, A. Bender and C.D. Roberts, Int. J. Mod. Phys. **A 10**, 3319 (1995).

[16] E.Z. Avakian, S.L. Avakian, G.V. Efimov and M.A. Ivanov, Fortsch. Phys. **38**, 611 (1990).

[17] P. Maris and C.D. Roberts, Phys. Rev. **C 58**, 3659 (1998).

[18] D.M. Asner *et al.* [CLEO Collaboration], "Hadronic structure in the decay $\tau \to \nu_\tau \pi^- \pi^0 \pi^0$ and the sign of the tau neutrino helicity," hep-ex/9902022.

[19] P. Jain and H.J. Munczek, Phys. Rev. **D 48**, 5403 (1993).

[20] C.J. Burden, L. Qian, C.D. Roberts, P.C. Tandy and M.J. Thomson, Phys. Rev. **C 55**, 2649 (1997).

[21] P. Maris, unpublished.

[22] C. D. Roberts, in *Quark Confinement and the Hadron Spectrum II*, edited by N. Brambilla and G. M. Prosperi (World Scientific, Singapore, 1997), pp. 224-230.

[23] M. Boglione and M.R. Pennington, Eur. Phys. J. **C 9**, 11 (1999).

[24] P. Maris, C.D. Roberts and P.C. Tandy, Phys. Lett. **B 420**, 267 (1998).

[25] I. Chappell, unpublished (1994).

[26] C.D. Roberts, R.T. Cahill, M.E. Sevior and N. Iannella, Phys. Rev. **D 49**, 125 (1994).

[27] J. Gasser and H. Leutwyler, Ann. Phys. **158**, 142 (1984).

[28] T. Blum and A. Soni [RBC Collaboration], "$K \to \pi\pi$ and ϵ'/ϵ using domain wall fermions," hep-lat/9909108.

[29] M. Neubert and B. Stech, Phys. Rev. **D 44**, 775 (1991).