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The mixing of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ and the search for the scalar glueball

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Abstract

For the first time a complete data set of the two-body decays of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ into all pseudoscalar mesons is available. The implications of these data for the flavour content for these three f_0 states is studied. We find that they are in accord with the hypothesis that the scalar glueball of lattice QCD mixes with the $q\overline{q}$ nonet that also exists in its immediate vicinity. We show that this solution also is compatible with the relative production strengths of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ in pp central production, $p\bar{p}$ annihilations and J/ψ radiative decays.

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The best estimate for the masses of glueballs comes from lattice gauge theory calculations which in the quenched approximation show [1] that the lightest glueball has $J^{PC} = 0^{++}$ and that its mass should be in the range $1.45 - 1.75$ GeV. While the lattice remains immature for predicting glueball decays, Amsler and Close [2] have noted that in lattice inspired models, such as the flux tube [3], glueballs will mix strongly with nearby $q\overline{q}$ states with the same J^{PC} [4]. This will lead to three isoscalar states of the same J^{PC} with predictable patterns of decay branching ratios [2, 5, 6]. Such mixing ideas have been applied recently to the three states in the glueball mass region - the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ [2, 6, 7, 8, 9].

While these papers at first sight differ in detail, nonetheless their conclusions share some common robust features. The flavour content of the states have the $n\overline{n}$ and $s\overline{s}$ in phase (SU(3) singlet tendency) for the $f_0(1370)$ and $f_0(1710)$, and out of phase (octet tendency) for the $f_0(1500)$ (that such a pattern is natural is discussed in ref. [10]). In general these mixings will negate the naive folklore that glueball decay branching ratios would be independent of the quark composition of the final state mesons after taking into account phase space effects, so called "flavour blind decays".

Recently ref.[8] has used some of the observed branching ratios as input to constrain the mixing pattern. The results here too agree with the generic structure found in refs. [2, 6, 7, 10]. Most recently, and subsequent to the above work, the WA102 collaboration has now published [11], for the first time in a single experiment, a complete data set for the decay branching ratios of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ to all pseudoscalar meson pairs. These data will be our point of departure. Using methods similar to those proposed in ref. [8] we investigate the implications of the WA102 data for the glueball-quarkonia content of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. This moves the debate forwards in the following ways:

- It highlights the sensitivity of the mixings to the input data.
- It exposes some assumptions, both explicit and implicit, in the analysis of ref. [8] that can be improved upon.
- It allows for the direct decay of glueballs into η and η', which were not manifested in leading order in ref.[8] even though there are reasons to suspect that they could be important [2, 12].

In order to unfold the production kinematics we use the invariant decay couplings (γ_{ij}) for the observed decays, namely we express the partial width (Γ_{ij}) as [2]

$$\Gamma_{ij} = \gamma_{ij}^2 |F_{ij}(\vec{q})|^2 S_p(\vec{q}) \tag{1}$$

where $S_p(\vec{q})$ denotes the phase space and $F_{ij}(\vec{q})$ are form factors appropriate to exclusive two body decays. Here we have followed ref. [2] and have chosen

$$|F_{ij}(\vec{q})|^2 = q^{2l} exp(-q^2/8\beta^2) \tag{2}$$

where l is the angular momentum of the final state with daughter momenta q and we have used $\beta = 0.5$ GeV/c [2]. The $f_0(1500)$ lies very near to threshold in the $\eta\eta'$ decay mode, therefore we have used an average value of q (190 MeV/c) derived from a fit to the $\eta\eta'$ mass spectrum.

The branching ratios measured by the WA102 experiment for the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ are given in table 1. The invariant decay couplings (γ_{ij}) are related to the relevant decay amplitudes M_{ij} by

$$\gamma_{ij}^2 = c_{ij}|M_{ij}|^2 \tag{3}$$

where c_{ij} is a weighting factor arising from the sum over the various charge combinations, namely 4 for $K\overline{K}$, 3 for $\pi\pi$, 2 for $\eta\eta'$ and 1 for $\eta\eta$ for isoscalar decays. If in the decay of some state the ratios of the decay amplitudes squared ($|M_{ij}|^2$) for the $\eta\eta/\pi\pi$ and $\eta\eta/K\overline{K}$ are simultaneously greater than unity, then this state cannot be a quarkonium decay (see fig. 3 of ref. [2] and also ref. [5]). Table 2 gives these ratios for the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, as abstracted using eqs.(1), (2) and (3). As can be seen the ratios for the $f_0(1710)$ argue either for non-$q\overline{q}$ content in this meson or for some further dynamical suppression of the $\pi\pi$ mode, say, as may occur for special values of parameters in some specific models [13].

In the $|G\rangle = |gg\rangle$, $|S\rangle = |s\bar{s}\rangle$, $|N\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$ basis, the mass matrix describing the mixing of a glueball and quarkonia can be written as follows [7]:

$$M = \begin{pmatrix} M_G & f & \sqrt{2}f \\ f & M_S & 0 \\ \sqrt{2}f & 0 & M_N \end{pmatrix}, \tag{4}$$

where $f = \langle G|M|S\rangle = \langle G|M|N\rangle/\sqrt{2}$ represents the flavour independent mixing strength between the glueball and quarkonia states. M_G, M_S and M_N represent the masses of the bare states $|G\rangle$, $|S\rangle$ and $|N\rangle$, respectively. Following refs. [2, 7, 8] we assume the mixing is strongest between the glueball and nearest $q\overline{q}$ neighbours. With the lattice (in the quenched approximation) predicting the glueball mass to be in the $1.45 - 1.75$ GeV region, this naturally led these papers to focus on the physical states $|f_0(1710)\rangle$, $|f_0(1500)\rangle$ and $|f_0(1370)\rangle$ as the eigenstates of M with the eigenvalues of M_1, M_2 and M_3, respectively. (An alternative picture could involve the states $f_0(1500)$, $f_0(1710)$, $f_0(2000)$; we do not discuss this in the present paper). The three physical states can be read as

$$\begin{pmatrix} |f_0(1710)\rangle \\ |f_0(1500)\rangle \\ |f_0(1370)\rangle \end{pmatrix} = U \begin{pmatrix} |G\rangle \\ |S\rangle \\ |N\rangle \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \begin{pmatrix} |G\rangle \\ |S\rangle \\ |N\rangle \end{pmatrix}, \tag{5}$$

where

$$U = \begin{pmatrix} (M_1 - M_S)(M_1 - M_N)C_1 & (M_1 - M_N)fC_1 & \sqrt{2}(M_1 - M_S)fC_1 \\ (M_2 - M_S)(M_2 - M_N)C_2 & (M_2 - M_N)fC_2 & \sqrt{2}(M_2 - M_S)fC_2 \\ (M_3 - M_S)(M_3 - M_N)C_3 & (M_3 - M_N)fC_3 & \sqrt{2}(M_3 - M_S)fC_3 \end{pmatrix} \tag{6}$$

with $C_{i(i=1,\,2,\,3)} = [(M_i - M_S)^2(M_i - M_N)^2 + (M_i - M_N)^2 f^2 + 2(M_i - M_S)^2 f^2]^{-\frac{1}{2}}$.

Ref.[8] considered the following three hadronic decay paths for the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$:

(i) the direct coupling of the quarkonia component of the three states to the final pseudoscalar mesons (PP) (fig. 1a),

(ii) the coupling through two intermediate gluons, $n\overline{n}(s\overline{s}) \to gg \to s\overline{s}(n\overline{n}) \to PP$ (fig. 1b), with r_1 representing the ratio of the effective coupling strength of this mode to that of the mode (i);

(iii) the flavour independent coupling of the glueball component $gg \to q_i\overline{q_i}$ with subsequent decay $q_i\overline{q_i} \to PP$ (fig. 1c); with r_2 representing the ratio of this mode to (i)

We propose that this is inconsistent. Specifically, the modes (ii) and (iii) as described above are what have already been accounted for in generating the mixed states and so r_1 and r_2 should be set to zero. This may be seen by comparing the definitions of eq.(6) with the corresponding expressions in perturbation theory (as e.g. eqs.(27-31) in ref [2] extended to second order). For example, eq.(27) of ref. [2] (where C_G denotes the normalisation factor)

$$(C_G)^{-1} f_0(G) = |G\rangle + \frac{|s\bar{s}\rangle\langle s\bar{s}|M|G\rangle}{M_G - M_{s\bar{s}}} + \frac{|n\bar{n}\rangle\langle n\bar{n}|M|G\rangle}{M_G - M_{n\bar{n}}} \tag{7}$$

may be written in the form of eq.(6), with $M_G \equiv M_1$, and $N \equiv n\bar{n}$, $S \equiv s\bar{s}$

$$\begin{aligned}(M_G - M_{n\bar{n}})(M_G - M_{s\bar{s}}) f_0(G) \quad &= \quad (M_1 - M_N)(M_1 - M_S) C_1|G\rangle + (M_1 - M_N) f C_1|s\bar{s}\rangle \\ &\quad + (M_1 - M_S)\sqrt{2} f C_1|n\bar{n}\rangle \end{aligned} \tag{8}$$

This shows how the coefficients y_1, z_1 are equivalent to the perturbation which is in turn driven by fig. 1c). Thus it is double counting to invoke this same figure with strength "r_2" to describe gg decays via $q\overline{q}$ intermediate states, having already used it to compute the mixing of those same $q\overline{q}$ in the Fock state. Similar remarks apply to the $s\overline{s} \to n\overline{n}$ mixing in second order perturbation theory and fig. 1b defined as r_1.

However, there are additional pathways that have not been allowed for in ref. [8]. First there is the role of $gg \to qq\overline{qq}$ as in fig. 1d. These may have important resonant contributions in the region below 1 GeV but are expected to be primarily a continuum in the 1.5 GeV region of interest here [14]; we shall approximate them by assuming flavour independent couplings. (In a more sophisticated analysis one could incorporate threshold effects in the PP channels that overlap the $qq\overline{qq}$ configurations [15]; we do not discuss this further in this first look). The resulting amplitudes can be obtained from eqs.(A4) of ref. [2] and have the same structure as those of (iii) above. Hence a non-zero r_2 is restored, though its interpretation differs from ref [8].

Finally, following ref [2, 12], we allow for

(iv) the direct coupling of the glue in the initial state to isoscalar mesons (i.e. $\eta\eta$ and $\eta\eta'$ decays). As in ref [2], we assume chiral symmetry such that the coupling to the $s\overline{s}$ content of the η, η' dominates and allow r_3 to be the ratio of mode (iv) to (i).

The three decay diagrams considered are shown in fig. 2a-c. Performing an elementary SU(3) calculation gives the reduced partial widths in table 3, where $\alpha = (\cos\phi - \sqrt{2}\sin\phi)/\sqrt{6}$, $\beta = (\sin\phi + \sqrt{2}\cos\phi)/\sqrt{6}$, and ϕ is the mixing angle of η and $\eta\prime$. The relevant expressions follow from appendix A in ref. [2] with $\rho = R = 1$ in the case of flavour independence of the direct couplings. The predicted branching ratios can then be calculated using eqs.(1) and (2).

We then perform a χ^2 fit based on the branching ratios given in table 1, where we have required that the matrix U in eq.(5) is unitary, which applies an additional 6 constraints to the fit. As input we use the masses of the $f_0(1500)$ and $f_0(1710)$. In this way eight parameters, M_G, M_N, M_S, M_3, f, r_2, r_3 and ϕ are determined from the fit. The parameters determined from the solution with the lowest χ^2 are presented in table 4 and the fitted branching ratios together with the χ^2 contributions of each are given in table 1.

The physical states $|f_0(1710)\rangle$, $|f_0(1500)\rangle$ and $|f_0(1370)\rangle$ are found to be

$$|f_0(1710)\rangle = 0.39|G\rangle + 0.91|S\rangle + 0.14|N\rangle, \tag{9}$$

$$|f_0(1500)\rangle = -0.69|G\rangle + 0.37|S\rangle - 0.62|N\rangle, \tag{10}$$

$$|f_0(1370)\rangle = 0.60|G\rangle - 0.13|S\rangle - 0.79|N\rangle. \tag{11}$$

It is interesting and non-trivial that the pattern of decays determines flavour mixing angles such that a state having an "octet" tendency is sandwiched between two states that have a "singlet" tendency. As noted above and elsewhere [10] this is a potential signal for G mixing with a $q\overline{q}$ nonet. The output masses for M_N and M_S are consistent with the $K^*(1430)$ being in the nonet and with the glueball mass being at the lower end of the quenched lattice range (see also [16, 17]). The mixing strength also is in accord with lattice estimates[7].

The elements in eqs.(9-11) form the matrix U as defined in eqs.(5,6). We have calculated the error on each element taking into account the correlated errors on their constituents which gives

$$\Delta U = \left(\begin{array}{ccc} 0.14 & 0.12 & 0.08 \\ 0.07 & 0.06 & 0.08 \\ 0.08 & 0.04 & 0.09 \end{array} \right). \tag{12}$$

This shows that the "singlet-octet-singlet" phase pattern is robust. The most sensitive probe of flavours and phases is in $\gamma\gamma$ couplings. In the spirit of ref. [6], ignoring mass-dependent effects, the above imply

$$\begin{array}{c} \Gamma(f_1(1710) \to \gamma\gamma) : \Gamma(f_1(1500) \to \gamma\gamma) : \Gamma(f_1(1370) \to \gamma\gamma) = \\ (5z_1 + \sqrt{2}y_1)^2 : (5z_2 + \sqrt{2}y_2)^2 : (5z_3 + \sqrt{2}y_3)^2 = 3.8 : 6.8 : 16.6. \end{array} \tag{13}$$

The $\gamma\gamma$ width of $f_0(1500)$ exceeding that of $f_0(1710)$ arises because the glueball is nearer to the N than the S. Contrast previous works where the G was nearer to (or even above) the S, in which case the $f_0(1500)$ has the smallest $\gamma\gamma$ coupling of the three states [6]. This shows how these $\gamma\gamma$ couplings have the potential to pin down the input pattern.

An interesting feature is the small value of the pseudoscalar mixing angle (ϕ); it is interesting that this value agrees with recent work that has $\phi(\eta) \neq \phi(\eta')$ [18]). We have checked that our results are not sensitive to allowing these angles to be independent. If instead we fix the value of ϕ to -19° degrees, the χ^2 of the fit increases from 3.0 to 7.7 and the results are given in tables 1 and 4 respectively. As can be seen the parameters of the fit are not very much affected.

We have also tried setting r_3 to zero: the χ^2 of the fit increases to 13.9 (see tables 1 and 4) and once again the parameters are not much altered.

Other authors have claimed that $M_G > M_S > M_N$ [7, 19, 20]. This scenario is disfavoured as, if in the fit we require $M_G > M_S > M_N$, the χ^2 increases to 57. In any event, we are cautious about such claims [7, 19, 20] as they are likely to be significantly distorted by the presence of a higher, nearby, excited $n\bar{n}$ state (N^*) such that $M_{N^*} > M_G > M_S$: the philosophy of dominant mixing with the nearest neighbours would then lead again to the "singlet - octet - singlet" scenario that we have found above. We defer detailed discussion to a more complete report.

Our preferred solution has two further implications for the production of these states in $p\overline{p}$ annihilations, in central pp collisions and in radiative J/ψ decays that are in accord with data. These are interesting in that they are consequences of the output and were not used as constraints.

The production of the f_0 states in $p\overline{p} \rightarrow \pi + f_0$ is expected to be dominantly through the $n\overline{n}$ components of the f_0 state, possibly through gg, but not prominently through the $s\overline{s}$ components. (The possible presence of hidden $s\overline{s}$ at threshold, noted by [21] is in general swamped by the above, and in any event appears unimportant in flight). The above mixing pattern implies that

$$\sigma(p\overline{p} \rightarrow \pi + f_0(1710)) < \sigma(p\overline{p} \rightarrow \pi + f_0(1370)) \sim \sigma(p\overline{p} \rightarrow \pi + f_0(1500)) \qquad (14)$$

Experimentally [22] the relative production rates are,

$$p\overline{p} \rightarrow \pi + f_0(1370) : \pi + f_0(1500)) \sim 1 : 1. \qquad (15)$$

and there is no evidence for the production of the $f_0(1710)$. This would be natural if the production were via the $n\overline{n}$ component.

For central production, the cross sections of well established quarkonia in WA102 suggest that the production of $s\overline{s}$ is strongly suppressed [23] relative to $n\overline{n}$. The relative cross sections for the three states of interest here are

$$pp \rightarrow pp + (f_0(1710) : f_0(1500) : f_0(1370)) \sim 0.14 : 1.7 : 1. \qquad (16)$$

This would be natural if the production were via the $n\overline{n}$ and gg components.

In addition, the WA102 collaboration has studied the production of these states as a function of the azimuthal angle ϕ, which is defined as the angle between the p_T vectors of the two outgoing protons. An important qualitative characteristic of these data is that the $f_0(1710)$ and $f_0(1500)$ peak as $\phi \rightarrow 0$ whereas the $f_0(1370)$ is more peaked as $\phi \rightarrow 180$ [24]. If the gg and $n\overline{n}$ components are produced coherently as $\phi \rightarrow 0$ but out of phase as $\phi \rightarrow 180$, then this pattern of ϕ dependence and relative production rates would follow; however, the relative coherence of gg and $n\overline{n}$ begs a dynamical explanation.

In J/ψ radiative decays, the absolute rates depend sensitively on the phases and relative strengths of the G relative to the $q\bar{q}$ component, as well as the relative phase of $n\bar{n}$ and $s\bar{s}$ within the latter. The general pattern though is clear. Following the discussion in ref. [6] we expect that the coupling to G will be large; coupling to $q\bar{q}$ with "octet tendency" will be suppressed; coupling to $q\bar{q}$ with "singlet tendency" will be intermediate. Hence the rate for $f_0(1370)$ will be smallest as the G interferes destructively against the $q\bar{q}$ with "singlet tendency". Conversely, the $f_0(1710)$ is enhanced by their constructive interference. The $f_0(1500)$ contains $q\bar{q}$ with "octet tendency" and its production will be driven dominantly by its G content. If the G mass is nearer to the N than to the S, as our results suggest, the G component in $f_0(1500)$ is large and cause the $J/\psi \to \gamma f_0(1500)$ rate to be comparable to $J/\psi \to \gamma f_0(1710)$.

In ref. [25], the branching ratio of $\mathrm{BR}(J/\psi \to \gamma f_0)(f_0 \to \pi\pi + K\bar{K})$ for the $f_0(1500)$ and $f_0(1710)$ is presented. Using the WA102 measured branching fractions [11] for these resonances and assuming that all major decay modes have been observed, the total relative production rates in radiative J/ψ decays can be calculated to be:

$$J/\psi \to f_0(1500) : J/\psi \to f_0(1710) = 1.0 : 1.1 \pm 0.4 \tag{17}$$

which is consistent with the prediction above based on our mixed state solution.

In these mixed state solutions, both the $f_0(1500)$ and $f_0(1710)$ have $n\bar{n}$ and $s\bar{s}$ contributions and so it would be expected that both would be produced in π^-p and K^-p interactions. The $f_0(1500)$ has clearly been observed in π^-p interactions: it was first observed in the $\eta\eta$ final state, although at that time it was referred to as the $G(1590)$ [26]. There is also evidence for the production of the $f_0(1500)$ in $K^-p \to K^0_S K^0_S \Lambda$ [27, 28]. The signal is much weaker compared to the well known $s\bar{s}$ state the $f_2'(1525)$, as expected with our mixings in eq.(10) and the suppressed $K\bar{K}$ decay associated with the destructive $n\bar{n} - s\bar{s}$ phase in the wavefunction.

There is evidence for the $f_0(1710)$ in the reaction $\pi^-p \to K^0_S K^0_S n$, originally called the $S^{*\prime}(1720)$ [29, 30]. One of the longstanding problems of the $f_0(1710)$ is that inspite of its dominant $K\bar{K}$ decay mode it was not observed in K^-p experiments [28, 31]. However, these concerns were based on the fact that initially the $f_0(1710)$ had $J = 2$. In fact, in ref. [32] it was demonstrated that if the $f_0(1710)$ had $J = 0$, as it has now been found to have, then the contribution in π^-p and K^-p are compatible. One word of caution should be given here: the analysis in ref. [32] was performed with a $f_0(1400)$ not a $f_0(1500)$ as we today know to be the case. As a further test of our solution, it would be nice to see the analysis of ref. [32] repeated with the mass and width of the $f_0(1500)$ and the decay parameters of the $f_0(1710)$ determined by the WA102 experiment.

In summary, based on the hypothesis that the scalar glueball mixes with the nearby $q\overline{q}$ nonet states, we have determined the flavour content of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ by studying their decays into all pseudoscalar meson pairs. The solution we have found is also compatible with the relative production strengths of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ in pp central production, $p\bar{p}$ annihilations and J/ψ radiative decays.

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Table 1: The measured and predicted branching ratios with the individual χ^2 contributions coming from the fits.

	Measured Branching ratio	All free Fitted	All free χ^2	ϕ = -19° Fitted	ϕ = -19° χ^2	r_3= 0 Fitted	r_3= 0 χ^2
$\frac{f_0(1370)\to\pi\pi}{f_0(1370)\to K\overline{K}}$	2.17 $\pm$ 0.9	2.13	0.001	2.1	0.004	2.25	0.007
$\frac{f_0(1370)\to\eta\eta}{f_0(1370)\to K\overline{K}}$	0.35 $\pm$ 0.21	0.41	0.1	0.01	2.6	0.01	2.6
$\frac{f_0(1500)\to\pi\pi}{f_0(1500)\to\eta\eta}$	5.5 $\pm$ 0.84	5.60	0.01	5.6	0.02	6.20	0.69
$\frac{f_0(1500)\to K\overline{K}}{f_0(1500)\to\pi\pi}$	0.32 $\pm$ 0.07	0.37	0.54	0.32	0.001	0.35	0.22
$\frac{f_0(1500)\to\eta\eta'}{f_0(1500)\to\eta\eta}$	0.52 $\pm$ 0.16	0.60	0.23	0.5	0.01	0.20	3.9
$\frac{f_0(1710)\to\pi\pi}{f_0(1710)\to K\overline{K}}$	0.20 $\pm$ 0.03	0.19	0.05	0.20	0.002	0.19	0.08
$\frac{f_0(1710)\to\eta\eta}{f_0(1710)\to K\overline{K}}$	0.48 $\pm$ 0.14	0.29	1.9	0.17	4.9	0.13	6.1
$\frac{f_0(1710)\to\eta\eta'}{f_0(1710)\to\eta\eta}$	$< 0.05(90\% cl)$	0.034	0.27	0.04	0.05	0.06	0.05

Table 2: The ratio of decay amplitudes squared.

	$\eta\eta/\pi\pi$	$\eta\eta/K\overline{K}$
$f_0(1370)$	0.74 ± 0.51	1.64 ± 0.96
$f_0(1500)$	0.68 ± 0.11	2.42 ± 0.64
$f_0(1710)$	7.9 ± 2.4	1.96 ± 0.64

Table 3: The theoretical reduced partial widths.

$\gamma^2(f_i \to \eta\eta')$	$2[2\alpha\beta(z_i - \sqrt{2}y_i) + 2\alpha\beta x_i r_3]^2$
$\gamma^2(f_i \to \eta\eta)$	$[2\alpha^2 z_i + 2\sqrt{2}\beta^2 y_i + r_2 x_i + 2\beta^2 x_i r_3]^2$
$\gamma^2(f_i \to \pi\pi)$	$3[z_i + r_2 x_i]^2$
$\gamma^2(f_i \to K\bar{K})$	$4[\frac{1}{2}(z_i + \sqrt{2}y_i) + r_2 x_i]^2$

Table 4: The solutions for the minimum χ^2.

Parameters	All Free	$\phi =$ -19°	$r_3 = 0$
χ^2	3.0	7.7	13.9
M_G (MeV)	1440 $\pm$ 16	1433 $\pm$ 19	1437 $\pm$ 15
M_S (MeV)	1672 $\pm$ 9	1668 $\pm$ 8	1672 $\pm$ 13
M_N (MeV)	1354 $\pm$ 28	1366 $\pm$ 25	1368 $\pm$ 29
M_3 (MeV)	1256 $\pm$ 31	1251 $\pm$ 18	1264 $\pm$ 14
f (MeV)	91 $\pm$ 11	95 $\pm$ 13	90 $\pm$ 11
ϕ (Deg)	-5 $\pm$ 4	-19	-25 $\pm$ 4
r_2	1.02 $\pm$ 0.14	0.92 $\pm$ 0.18	0.95 $\pm$ 0.12
r_3	1.04 $\pm$ 0.24	0.77$\pm$ 0.26	0

Figures

Figure 1: Possible Decays to Pseudoscalar meson pairs (PP). a) The direct coupling of the $q\bar{q}$ to the PP pair, b) the coupling of the $q\bar{q}$ to PP via intermediate gluons, c) the coupling of the glueball component to PP and d) the direct coupling of gluons to isoscalar mesons.

Figure 2: The Decays to Pseudoscalar meson pairs (PP) considered in this analysis. a) The coupling of the $q\bar{q}$ to the PP pair, b) the coupling of the glueball component to PP and c) the direct coupling of gluons to the gluonic component of the final state mesons.

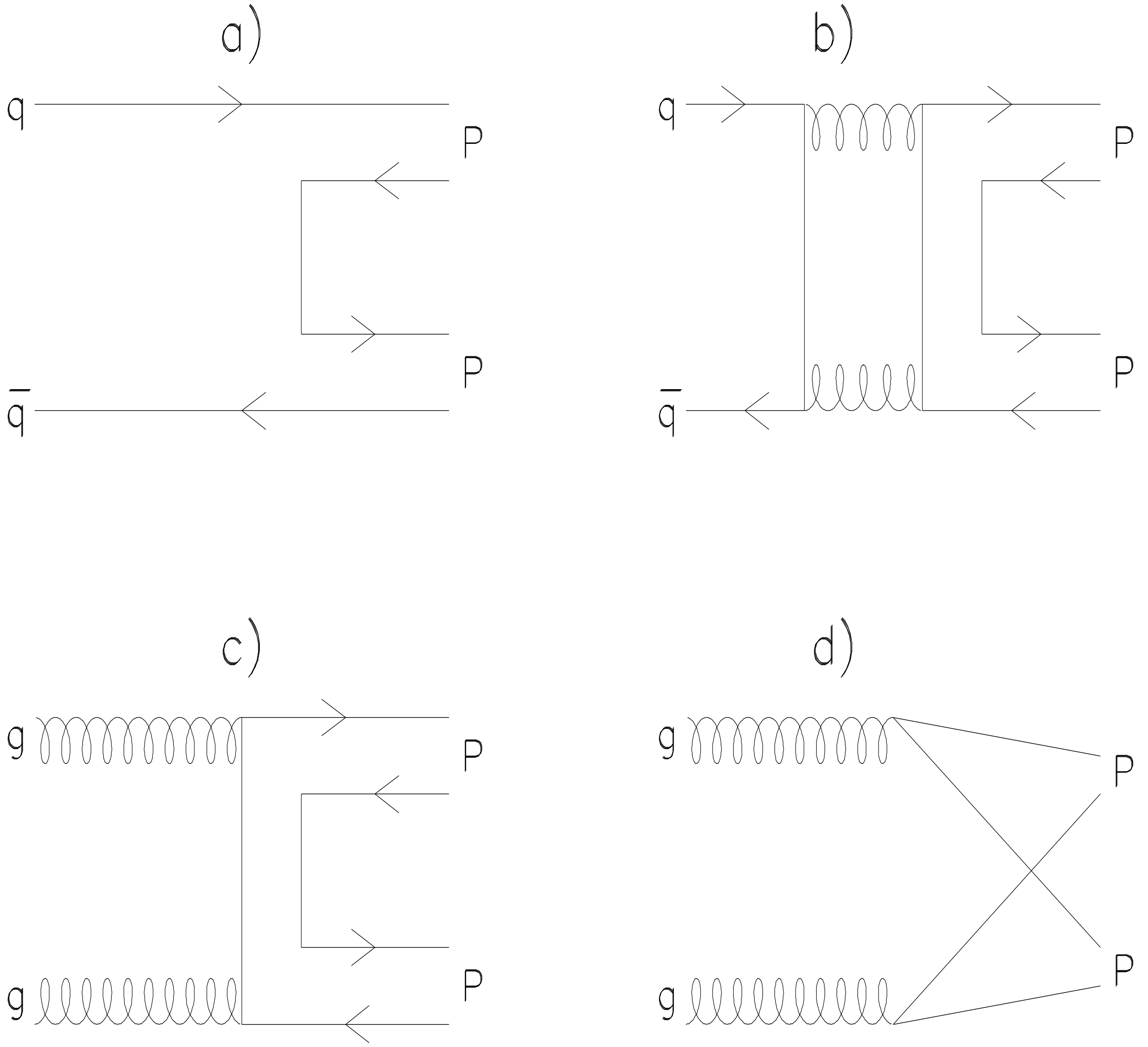

Figure 1

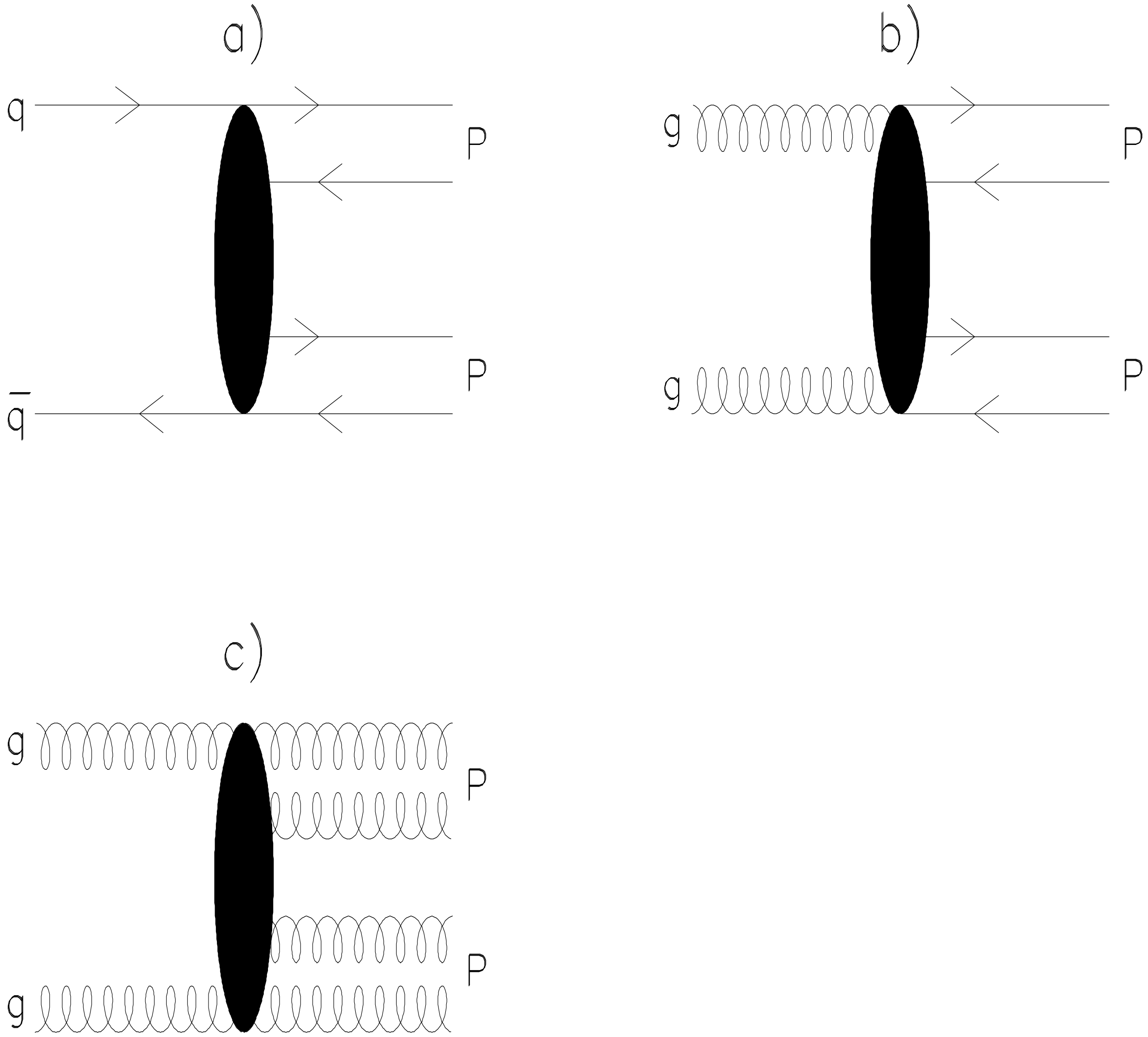

Figure 2