Exclusive hadron production in e^+e^- collisions up to the J/ψ energy.

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Abstract **The predictions are presented for the diagonal and transition form factors of light hadrons in the time-like region up to the production threshold of an open charm quantum number. The comparison with existing data on the decays of J/ψ and $\psi(2S)$ mesons into such hadrons shows that some new resonance structures may be present in the mass range between 2 GeV and the J/ψ mass. Searching them may help in a better understanding of the mass spectrum in quark models, and in revealing the details of the three-gluon mechanism of the OZI rule breaking.**

There are intentions to study the energy range of e^+e^- annihilation in the interval of the center-of-mass energy from $2E = 1.5$ GeV up to $m_{J/\psi}$ using the collider VEPP-4M [1]. The BEPC e^+e^- collider team has also a plan to study some exclusive channels in the energy range from 2 to 5 GeV [2,3]. This raises the question of comparison of the results of existing analysis of the diagonal and transition form factors of light hadrons in the energy range between 1 and 2 GeV [4,5] with the data now existing at the J/ψ [6] and $\psi(2S)$ [3,6] masses. Here we perform this task [7], in order to uncover possible surprises that might be revealed in future experiments.

The Okubo-Zweig-Iizuka (OZI) rule violating decays of the $c\bar{c}$ quarkonia into the light hadrons are divided into two very different classes. The isovector states $\pi^+\pi^-$, $\omega\pi^0$, $\rho\eta$ and $\rho^0\pi^+\pi^-$ are produced predominantly via the one photon (γ) intermediate state. The three-gluon (ggg) contribution which violates the conservation of isospin should be suppressed. Indeed, in the $\pi^+\pi^-$ channel, the ratio of the coupling constant due to three gluons to that due to one photon is estimated as

$$\frac{|a_\pi^{(ggg)}|}{|a_\pi^{(\gamma)}|} \sim \frac{m_d - m_u}{Q}\left(\frac{\alpha_s}{\pi}\right)^3 \frac{f_{J/\psi}}{4\pi\alpha|F_\pi(m^2_{J/\psi})|}, \tag{1}$$

where $\alpha = 1/137$, $\alpha_s \simeq 0.2$ is the QCD coupling constant, and $f_{J/\psi}$ enters the expression for the leptonic width of the J/ψ in a usual way:

$$\Gamma_{J/\psi\to e^+e^-} = \frac{4\pi\alpha^2}{3f^2_{J/\psi}} m_{J/\psi}. \tag{2}$$

Inserting $m_d - m_u \simeq 3$ MeV, choosing conservatively $Q \sim m_\pi$, and taking the vector dominance model (VDM) expression

$$F_\pi^{(\mathrm{VDM})}(s) = \frac{m_\rho^2}{m_\rho^2 - s} \tag{3}$$

for the pion form factor, one gets the figure of 10^{-2} for above ratio. Similar estimate holds for other isovector channels cited above. The amplitude with $gg\gamma$ in intermediate state is also expected to be suppressed [8]. The production amplitude of the isoscalar states includes the superposition of the one photon and ggg amplitudes. The production amplitude of strange mesons includes the superposition of both the isovector and isoscalar amplitudes. First we will compare the data on the J/ψ decays with the predictions of the corresponding VDM expression assuming the zero-width approximation and then to more sophisticated amplitudes which incorporate the complex mixing of mesons from the ground state nonet with the heavier primed resonances [4,5]. The expression for the isovector formfactor can be written in this case as

$$F_f(s) = \left(\frac{m_\rho^2}{f_\rho}, \frac{m^2_{\rho'_1}}{f_{\rho'_1}}, \frac{m^2_{\rho'_2}}{f_{\rho'_2}}\right) G^{-1}(s) \begin{pmatrix} g_{\rho f} \\ g_{\rho'_1 f} \\ g_{\rho'_2 f} \end{pmatrix}, \tag{4}$$

where $f = \pi^+\pi^-$, $\omega\pi^0$ and $\eta\pi^+\pi^-$; s is the total center-of-mass energy squared, $\alpha = 1/137$. For a purpose of uniformity of the expression Eq. (4) in the case of the $\pi^+\pi^-$ channel the contribution of the $\rho\omega$ mixing is omitted. It can be found in Ref. [4]. The matrix of inverse propagators in Eq. (4) looks as

$$G(s) = \begin{pmatrix} D_\rho & -\Pi_{\rho\rho'_1} & -\Pi_{\rho\rho'_2} \\ -\Pi_{\rho\rho'_1} & D_{\rho'_1} & -\Pi_{\rho'_1\rho'_2} \\ -\Pi_{\rho\rho'_2} & -\Pi_{\rho'_1\rho'_2} & D_{\rho'_2} \end{pmatrix}. \tag{5}$$

It contains the inverse propagators of the unmixed states $\rho_i = \rho(770)$, ρ'_1, ρ'_2,

$$D_{\rho_i} \equiv D_{\rho_i}(s) = m^2_{\rho_i} - s - i\sqrt{s}\Gamma_{\rho_i}(s), \tag{6}$$

where $\Gamma_{\rho_i}(s)$ are the energy dependent widths whose expressions are given in [4], and the nondiagonal polarization operators

$$\Pi_{\rho_i\rho_j} = \mathrm{Re}\Pi_{\rho_i\rho_j} + i\mathrm{Im}\Pi_{\rho_i\rho_j}$$

describing the mixing. Their real parts are found from the fits [4] to be consistent with zero while imaginary parts are given by the unitarity relation, see Ref. [4] for more detail.

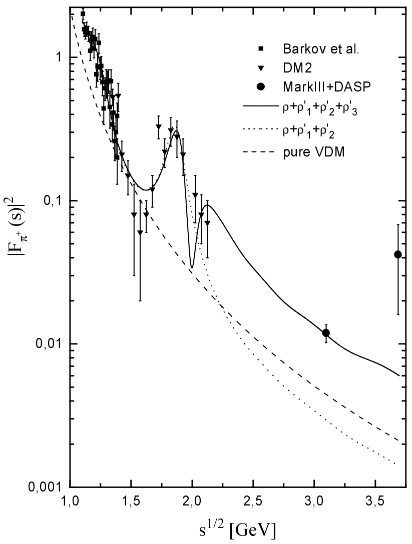

FIG. 1. The pion form factor. The data are from Barkov et al. [18], DM2 [19], MarkIII [9], DASP [10].

Let us present our findings first for the decay channels with the pair of pseudoscalar mesons. The modulus squared of the pion form factor expressed through the ratio of partial widths,

$$|F_\pi(m^2_{J/\psi})|^2 = 4\frac{\Gamma(J/\psi \to \pi^+\pi^-)}{\Gamma(J/\psi \to e^+e^-)}, \qquad (7)$$

is $(11.9 \pm 1.5 \pm 0.9) \times 10^{-3}$ [9] [or slightly lower figure of $(9.8 \pm 1.5) \times 10^{-3}$, according to the averaged value of the $\pi^+\pi^-$ branching ratio found in [6]] and was already mentioned to be remarkably large [8]. The VDM estimate according to Eq. (3) (see the dashed curved in Fig. 1) amounts to a figure of 4.3×10^{-3}. In the case of $\psi(2S)$ the pion form factor can be evaluated with the formula similar to Eq. (7) and gives, using the earlier DASP data [10], the figure of $|F_\pi(m^2_{\psi(2S)})|^2 = (36 \pm 23) \times 10^{-3}$. This is especially surprising since shows, guided by the central figure, the rise of the form factor with the energy increase, but, certainly, experimental error is too large. Using a more realistic amplitude Eq. (4) which includes the $\rho'_{1,2}$ resonances with the parameters obtained recently [4], we plot the corresponding curve with the dotted line in Fig. 1. In this case the curve goes four times as low as compared to the experimental value at the J/ψ mass. This is puzzling, since the one photon contribution is the only way to explain the decay J/$\psi \to \pi^+\pi^-$.

The above theoretical inconsistencies of the $\pi^+\pi^-$ channel strongly suggest that something new may happen at the energies between 2 GeV and the mass of J/ψ, where the data are almost absent. As an illustration, we add the resonance $\rho(2150)$ with the quantum numbers $I^G(J^{PC}) = 1^+(1^{--})$ documented in the full listings of Review of Particle Physics (RPP) [6], ignoring, for nothing is better, the possible energy dependence of its partial widths and the mixing with other ρ-like resonances. Taking the mass $m_{\rho'_3} = 2010$ MeV, the width $\Gamma_{\rho'_3} = 260$ MeV, the ratio of coupling constants $g_{\rho'_3\pi\pi}/f_{\rho'_3} = 0.08$, and slightly varying, within the error bars, the parameters of the $\rho'_{1,2}$ resonances found in Ref. [4], one obtains the curve shown with the solid line in Fig. 1. One can see that the knowledge of the spectrum of still unknown isovector resonances (if any) above 2 GeV is crucial for both the understanding of the behavior of the pion form factor (and some other form factors, too, see below) and for establishing the limits to applicability of the generalized VDM.

In general, the $K\bar{K}$ coupling of a C-odd quarkonium $J/\psi = c\bar{c}$ is represented in the form

$$g_{\mathrm{J}/\psi\mathrm{K\bar{K}}} = a_K^{(ggg)} - \frac{4\pi\alpha}{f_{\mathrm{J}/\psi}}(\pm F_K^{(1)} + F_K^{(0)}), \qquad (8)$$

where $a_K^{(ggg)}$ being, in general, a complex number, represents the pure isoscalar contribution of the three gluons; $F_K^{(I)} \equiv F_K^{(I)}(m^2_{\mathrm{J}/\psi})$ is the kaon electromagnetic form factor with the given isospin $I = 0, 1$ taken at the J/ψ mass [11]. The leptonic coupling constant $f_{\mathrm{J}/\psi}$ is expressed through leptonic partial width by the expression Eq. (2). The K^+K^- and K_LK_S decay rates are distinguished by the sign of isovector contribution, so that the ratio of $|F_K^{(1)}(m^2_{\mathrm{J}/\psi})|$ extracted from the data [9], to the VDM estimate

$$|F_{K^+}^{(1)(\mathrm{VDM})}(m^2_{\mathrm{J}/\psi})| = \frac{1}{2}\frac{m^2_\rho}{(m^2_{\mathrm{J}/\psi} - m^2_\rho)} = 0.033 \qquad (9)$$

is found to be 2, 1, 2/3 for the relative phase of the $I = 0$ and $I = 1$ contributions $\theta = 62°, 22°, 0°$, respectively. Note that the latter case gives the lower bound to the isovector contribution. However, the simple VDM amplitude fails to describe the data on the reaction $e^+e^- \to K^+K^-$ in the energy range 2E=1.1−2 GeV; see Fig. 2. On the other hand, the isovector part of the kaon form factor extracted from the fit which includes the contributions of heavier resonances $\rho'_{1,2}$, $\omega'_{1,2}$, and $\varphi'_{1,2}$ with the parameters found in Ref. [5], can be matched with isovector contribution extracted from the J/ψ data, provided the relative phase is $\theta = 22°$. Accidently, at the J/ψ mass, the absolute values of the isovector kaon form factor in the simple VDM and in our fit [5] turn out to be coincident. In the meantime, the phase relations in the above models are completely different. Specifically, one has

$$F_K^{(0)}(m^2_{\mathrm{J}/\psi}) = (6.5 - 6.3i) \times 10^{-3},$$
$$F_K^{(1)}(m^2_{\mathrm{J}/\psi}) = (3.1 - 1.3i) \times 10^{-2}, \qquad (10)$$

with the set of parameters found in Ref. [5]. Note that the modulus of the three-gluon coupling constant satisfies

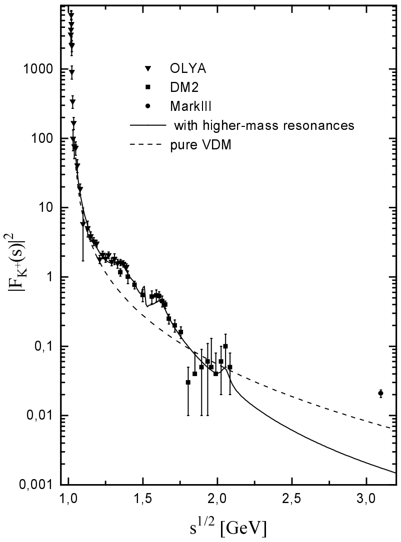

FIG. 2. The charged kaon form factor. The experimental point at the J/ψ mass is given by the authors of Ref. [9] upon neglecting the three-gluon contribution. The data are from OLYA [20], DM2 [21], MarkIII [9]. The solid curve is drawn upon taking into account the contributions of higher mass resonances $(\rho_1' + \omega_1' + \varphi_1') + (\rho_2' + \omega_2' + \varphi_2')$, with the parameters found in [4,5].

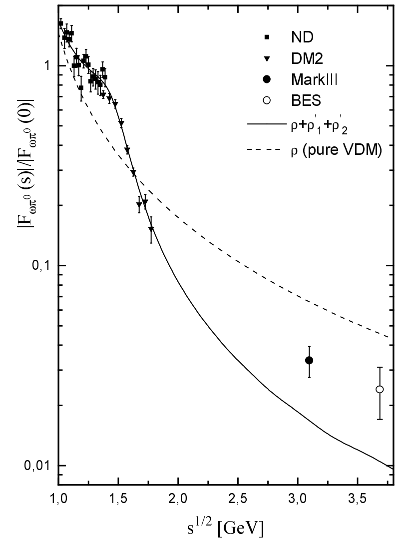

FIG. 3. The $\omega\pi^0$ form factor. The data are recalculated from the cross section data of ND [22], DM2 [23]; the BES data are from [3].

the relation $|a_K^{(ggg)}| \geq 0.9|g_0|$ regardless the relative phase between the three-gluon contribution and isoscalar part of the one photon one. Here the numerical factor of 0.9 comes from the numerical value of the isoscalar kaon form factor given in Eq. (10), and g_0 is the coupling constant of J/ψ to $K\bar{K}$ in the $I = 0$ state. Since one can hardly imagine the mechanism of enhancement of the isoscalar form factor by an order of magnitude in comparison with that given in Eq. (10), we see that the greater part of the isoscalar coupling constant is due to the three-gluon contribution. There are no reasons to neglect the latter and attribute all the $K\bar{K}$ branching ratio of the J/ψ solely to the one photon mechanism, as it was assumed in Ref. [9].

Now turn to the vector and pseudoscalar final states [12–14]. The ratio of the absolute values of the $\omega\pi^0$ form factors is expressed through the measured branching ratios as [12,13]

$$\frac{|F_{\omega\pi^0}(m^2_{J/\psi})|}{|F_{\omega\pi^0}(0)|} = \left[\frac{\alpha}{3}\left(\frac{q_{\gamma\pi^0}}{q_{\omega\pi^0}}\right)^3 \frac{m_{J/\psi}}{\Gamma(\omega \to \gamma\pi^0)} \times \frac{\Gamma(J/\psi \to \omega\pi^0)}{\Gamma(J/\psi \to \mu^+\mu^-)}\right]^{1/2}. \tag{11}$$

The VDM evaluation of the above ratio gives a figure of 0.0659 which is by a factor of two greater than the experimentally measured figure of 0.0335 ± 0.0059 [13]. On the other hand, the inclusion of the $\rho'_{1,2}$ resonances [4] interfering destructively with the $\rho(770)$ tail at energies above 2 GeV results in the calculated figure to be twice as low as experimentally measured. See the curve in Fig. 3. The result of the calculation of an analogous ratio for the $\rho\eta$ final state is shown in Fig. 4.

Finally, the form factor of the $\rho^0\pi^+\pi^-$ final state which enters the partial width of the J/ψ as

$$\Gamma(J/\psi \to \rho^0\pi^+\pi^-) = 12\pi\alpha\frac{\Gamma(J/\psi \to \mu^+\mu^-)}{m_{J/\psi}} \times \left|F_{\rho^0\pi^+\pi^-}(m^2_{J/\psi})\right|^2 \times W_{\pi^+\pi^-\pi^+\pi^-},(m^2_{J/\psi}) \tag{12}$$

where $W_{\pi^+\pi^-\pi^+\pi^-}$ is the phase space volume of the $2\pi^+2\pi^-$ state given in [4], analogously for the $\psi(2S)$, is plotted in Fig. 5. The VDM estimate in this case is

$$F_{\rho^0\pi^+\pi^-}(s) = \frac{2g_{\rho\pi\pi}m^2_\rho}{m^2_\rho - s}, \tag{13}$$

where the relation among the coupling constants $g_{\rho^0\rho^0\pi^+\pi^-} = 2g^2_{\rho\pi\pi}$ resulting from the vector current conservation is taken into account, together with the neglect of the bremstrahlung-type diagrams. See Ref. [4] for some details of approximations made for the $\rho\rho\pi^+\pi^-$ coupling. Both curves go far below the J/ψ and $\psi(2S)$ data.

Note that the $\rho^0\pi^+\pi^-$ contribution was not isolated in the total $\pi^+\pi^-\pi^+\pi^-$ data sample at the J/ψ mass

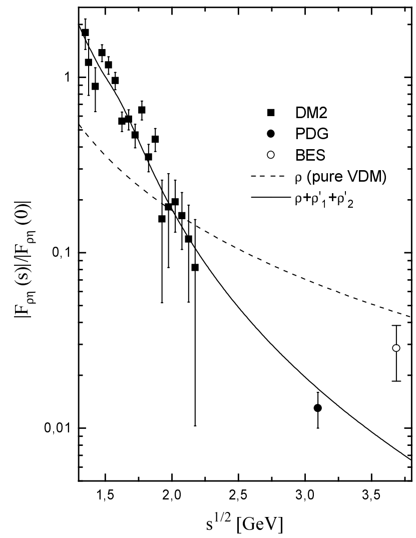

FIG. 4. The $\rho\eta$ form factor. The DM2 data are recalculated from the cross section data of [24], PDG [6], the BES data are from [3].

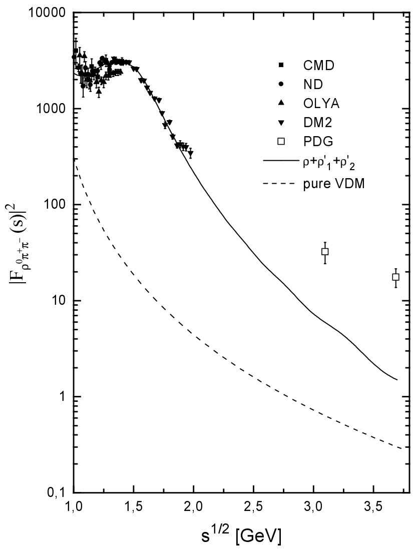

FIG. 5. The $\rho\pi^+\pi^-$ form factor squared. The data are recalculated from the cross section data of CMD [25], ND [22], OLYA [26], DM2 [23]; MARKI [15,16].

[15]. However, such an isolation was implemented at the $\psi(2S)$ mass, and the $\rho^0\pi^+\pi^-$ contribution was found to be 93% [16] of the total number of $\pi^+\pi^-\pi^+\pi^-$ events. Since one cannot foresee any reason why the situation, in this respect, at the J/ψ could differ from the $\psi(2S)$, we simply insert $B(J/\psi \to \pi^+\pi^-\pi^+\pi^-)$ in place of $B(J/\psi \to \rho^0\pi^+\pi^-)$, in order to find the $\rho^0\pi^+\pi^-$ transition form factor at the J/ψ mass. Since in almost all cases the curves in Fig. 1–5 go well below the J/ψ data points, one can see that some isovector resonance structures with the masses above 2 GeV interfering strongly with those already included are likely to be present. The example of the $\pi^+\pi^-$ channel shows that the fit of the data with the J/ψ data point included is improved with the ρ_3' resonance being taken into account [17]. Their isoscalar partners are also rather probable. They could manifest themselves in the channels of e^+e^- annihilation into $\omega\eta$, $\omega\eta'$, $\rho\pi$, $\omega\pi^+\pi^-$ etc and in the decay channels which include strange particles. All this suggests that the energy region above 2 GeV of e^+e^- annihilation is interesting from the point of view of elucidating the spectrum of states with the masses in this range and for establishing the detailed form (modulus and phase) of the three-gluon coupling with different states including its dependence on energy. To gain an impression of what the typical cross section magnitudes might be, we give the calculated figures at the energy $\sqrt{s} = 2.5$ GeV. In the case of the final states $\pi^+\pi^-$, K^+K^-, $\omega\pi^0$, $\rho^0\eta(\pi^+\pi^-\eta)$, and $\pi^+\pi^-\pi^+\pi^-$ they are, respectively, 0.03, 0.02, 0.04, 0.04, and 0.6 nanobarns.

[1] L. M. Kurdadze, private communication.
[2] Xu Gou-fa, private communication.
[3] S. L. Olsen, these Proceedings;
F. A. Harris, hep-ex/9903036;
S. F. Tuan, hep-ph/9903322.
[4] N. N. Achasov and A. A. Kozhevnikov, Phys. Rev. D**55**, 2663 (1997); hep-ph/9609216.
[5] N. N. Achasov and A. A. Kozhevnikov, Phys. Rev. D**57**, 4334 (1998); hep-ph/9703397.
[6] C. Caso *et al.* (Particle Data Group), The European Physical Journal **C3**, 1 (1998).
[7] N. N. Achasov and A. A. Kozhevnikov, Phys. Rev. D**58**, 097502 (1998); hep-ph/9801308.
[8] J. Milana, S. Nussinov and M. G. Olsson, Phys. Rev. Lett., **71**, 2533 (1993).
[9] R. M. Baltrusaitis *et al.*, Phys. Rev. D**32**, 566 (1985).
[10] R. Brandelik *et al.*, Z. Phys. C**1**, 233 (1979).
[11] The arguments in favor of neglecting the one-photon coupling to the neutral kaon pair put forward in Ref. [8] would be fulfilled only in the case of exact degeneracy of the $\rho(770)$, $\omega(782)$, and $\varphi(1020)$ masses. In reality, however, one obtains the ratio $|F_{K^+}(m^2_{J/\psi})|/|F_{K^0}(m^2_{J/\psi})|$ equal to 4.5 and 1.7 in the simple VDM and in the VDM with inclusion of heavier resonances [4,5], respectively.
[12] R. M. Baltrusaitis *et al.*, Phys. Rev. D**32**, 2883 (1985).
[13] D. Coffman *et al.*, Phys. Rev. D**38**, 2695 (1988).
[14] J. Jousset *et al.*, Phys. Rev. D**41**, 1389 (1990).
[15] B. Jean-Marie *et al.* Phys. Rev. Lett., **36**, 291 (1976).

[16] W. Tanenbaum *et al.* Phys. Rev. D**17**, 1731 (1978).
[17] Because of the lack of data above 2 GeV, the coupling constants of the ρ_3' resonance with specific channels are unknown, so we do not include it in other channels except the $\pi^+\pi^-$ one. The procedure of extracting the ρ_3' couplings, with the only J/ψ data points at hand, would appear to be speculative, since the χ^2 criterion determined in that case essentially by the low energy data points, would not put any serious restrictions on magnitudes of these couplings.
[18] L. M. Barkov *et al.*, Nucl. Phys. **B256**, 365 (1985).
[19] DM2 Collaboration, D. Bisello *et al.*, Phys. Lett. B**220**, 321 (1989).
[20] P. M. Ivanov *et al.* Phys. Lett **107B**, 297 (1981).
[21] DM2 Collaboration, D. Bisello *et al.* Z. Phys. C**39**, 13 (1988).
[22] S. I. Dolinsky *et al.*, Phys. Rep. **202**, 99 (1991).
[23] L. Stanco, in *Hadron 91*, Proceedings of the International Conference on Hadron Spectroscopy, College Park, Maryland, edited by S. Oneda and D. C. Peaslee (World Scientific, Singapore, 1992) p. 84.
[24] DM2 Collaboration, A. Antonelli *et al.*, Phys. Lett. B**212**, 133 (1988).
[25] L. M. Barkov *et al.* Yad. Fiz. **47**, 393 (1988).
[26] L. M. Kurdadze *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 432 (1988).