Helicity Amplitudes for Final State Particle with non-zero Spin

Peppino Giarritta, University of Zurich

Crystal Barrel Jamboree Meeting / March 1,1999

Contents

- Helicity Formalism
- Problem of Calculating the Transition Probability (Ex. pp$\rightarrow$ $b_1\,\pi$, $b_1 \rightarrow \omega\pi$)
- The Correct Transition Amplitude
- Conclusion

Helicity Formalism

1) Two body decay $(P,J,M) \rightarrow (p_1,s_1,\lambda_1) \otimes (p_2,s_2,\lambda_2)$

$$\boxed{A_{M;\,\lambda 1\,\lambda 2}(\theta,\phi) = D^{J}_{M\,\lambda}{}^{*}(\theta,\phi)\; A_{\lambda;\,\lambda 1\,\lambda 2}(0,0)}$$

(Eq. 1)

$\lambda = \lambda_1 - \lambda_2$

$A_{\lambda;\,\lambda 1\,\lambda 2}(0,0) = \langle J\lambda;\, LS\, \mathbf{0}\lambda\rangle$

$\qquad \langle J\lambda;\, s_1 s_2 \lambda_1 - \lambda_2\rangle\; BW_L$

$D^{J}_{M\,\lambda}(\theta,\phi) = D^{J}_{M\,\lambda}(\phi,\theta,0)$

((J+1)-dimensional representation of the frame rotation, see QM)

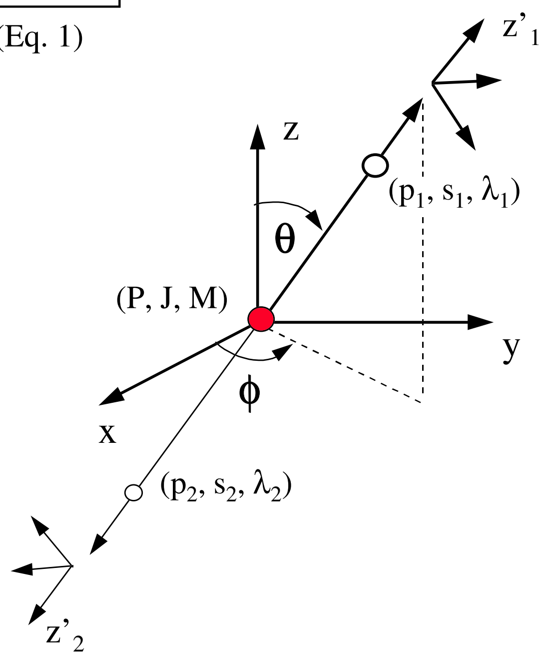
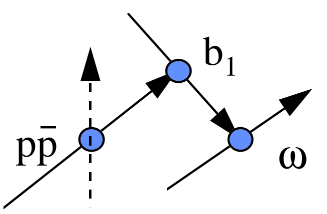


Fig 1

2) Decay Chain, Ex: $pp \rightarrow b_1\,\pi,\; b_1 \rightarrow \omega\pi,\;\; \omega \rightarrow \pi\,\gamma$



$\gamma\ (\lambda_5)$

Fig 2: Subsequent frame transformation

$$A = A_{M;\,\lambda 5\,0} = A^{JLS}{}_{M;\,\lambda 1\,0}(pp) * A^{jls}{}_{\lambda 1;\,\lambda 3\,0}(b_1) * A^{111}{}_{\lambda 3;\,\lambda 5\,0}(\omega) \qquad \text{(Eq. 2)}$$

3) Transition Probability

$$\boxed{w_D = \mathrm{Tr}\,\rho_f = \mathrm{Tr}\,(T\,\rho_0\,T^{+})\;;\quad T = \alpha_i A^i} \qquad \text{(Eq. 3)}$$

Problem of Calculating the Transition Probability

1) Ex: $pp(1^{--}) \rightarrow b_1 \pi$ (L=0); $b_1 \rightarrow \omega\pi$ (l=0); $\omega \rightarrow \pi\gamma$

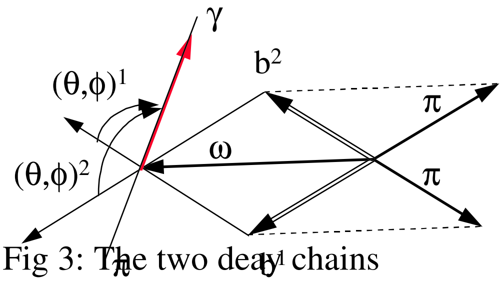


Fig 3: The two decay chains

2 combinatorial possibilities:
b^1: $A^1_{M;\,\lambda\,0}$,
b^2: $A^2_{M;\,\lambda\,0}$

$$T_{M;\,\lambda\,0} = A^1_{M;\,\lambda\,0} + A^2_{M;\,\lambda\,0}$$

2) MC Simulation vs. QM

$$w_{MC} = w_{PS} \times \varepsilon \times w_D$$

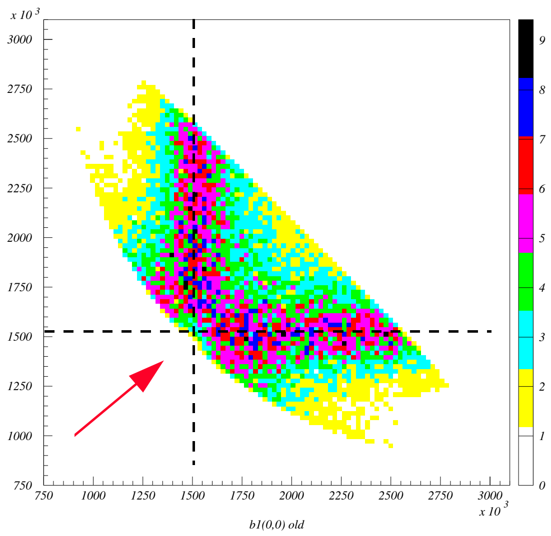


Fig 4: Simulated Dalitz plot

MC: There is a destructive behavior at the crossing of the two b_1 bands

(contradiction)

QM: Because of ***L*=0** and ***l*=0** one would expect **no angular dependence** (interference only determined by the dynamics, eg BW)*

* $\mathbf{J} = \mathbf{S}_\omega$; $1^{--} \rightarrow 1^{+-}\,0^{-+}$, $1^{+-} \rightarrow 1^{--}\,0^{-+}$, $1^{--} \rightarrow 1^{--}\,0^{-+}$

The Correct Transition Amplitude

The frame (assumed in eq.2) of the final state (ω) depends on the decay chain!
The difference between the chains is a rotation around the z axis which translates into an artificial phase (see fig 5).

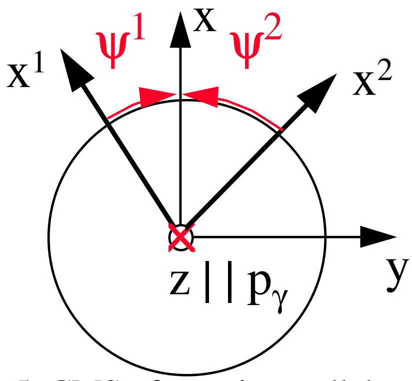


Fig 5: CMS of ω, z is parallel to the γ direction ω (see also fig 2)

Frames:

Σ^1: (x^1, y^1, z)

Σ^2: (x^2, y^2, z)

Σ: (x, y, z) reference system

Solution: For each event and each chain the angle ψ has to be calculated and the transition amplitude has to be corrected by the phase Exp(i $\lambda_f\, \omega\psi^j$) (see Eq. 4).

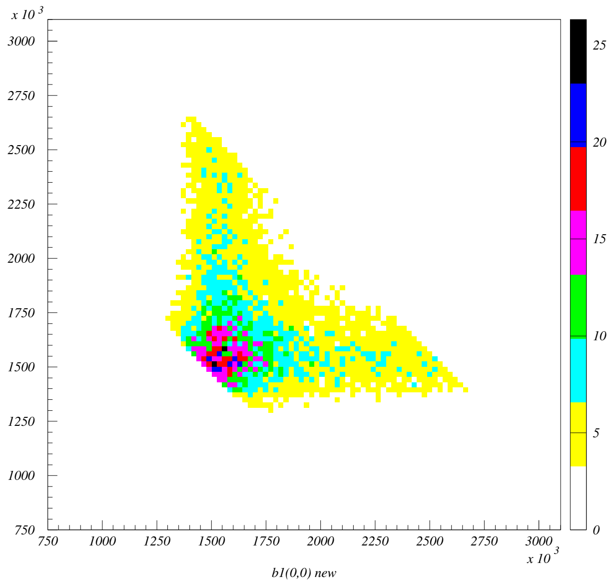


Fig 6: Simulated Dalitz plot (corrected Transition)

$$\boxed{A^j \rightarrow A^j\, \mathrm{Exp}(\, i\, \lambda_f\, \psi^j)} \quad \text{(Eq. 4)}$$

Conclusion

- No correction is needed for **scalar** final state (λ=0 !)
- For final states **with spin** the alignment of the final state frame has to be considered rigorously , the later can change the phase of the transition amplitude.
  In order to compensate a rotation around z, the phase correction $exp(i \lambda_f \psi)$ has to be applied (see eq. 4) .
- Nevertheless the helicity formalism is correct, if correctly applied!

Further Reading (Selection)

Formalism

M.Jacob and G.C.Wick, Ann. of Phys. 7(1959)404

J.D.Richman, An Experimenter's Guide to the Helicity Formalism, Pasadena, 1968, CALT-68-1148

J.D.Hansen et al., Formalism and Assumptions Involved in Partial-Wave Analysis of three-Meson Systems, NPB 81(1974) 403

Computing

C. Amsler et J.C.Bizot, 1983, Comput.Phys.Commun. 30,21 [about *SPIN*]

R. McCrady, Using Mathematica to Compute Helicity Amplitudes 1996, CB-Note 298

Th. Degener, PhD Thesis, Bochum1998 and Program TARA.C++

P. Giarritta PhD Thesis (pp $\rightarrow \omega\pi\pi$, $\rightarrow \omega\eta\pi$), Zurich, in preparation

Covariant Formalism

I. Augustin, Rarita-Schwinger Formalism (based on the Zupancic Lectures) 1991, CB-Note

V. Filippini, A.Fontana and A.Rotondi, Covariant Spin Tensors in Meson Spectroscopy, Phs.Rev D 51(1995)2247

V. Filippini Spin Formalismus in p bar-p Annihilation, LEAP98

S.U.Chung General Formulation of Covariant Helicity-Coupling Amplitudes Phys.Rev. D57 (1998) 431