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Multichannel Interaction Analysis of Scalar Mesons Below 1800 MeV

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ABSTRACT

Properties of scalar–isoscalar mesons in a mass range from $\pi\pi$ threshold up to 1800 MeV are analysed using an unitary model with separable interactions in three decay channels: $\pi\pi$, $K\overline{K}$ and an effective $2\pi 2\pi$. Different solutions are obtained by fitting $\pi\pi$ and $K\overline{K}$ data. Analytical structure of the meson-meson multichannel amplitudes is studied with a special emphasis on the important role played by the S-matrix zeroes. The dependence of the positions of S-matrix singularities on the interchannel coupling strength is investigated. Poles, located in the complex energy plane not too far from the physical region, are interpreted as scalar resonances: a wide $f_0(500)$, a narrow $f_0(980)$ and a relatively narrow $f_0(1400)$. In all our solutions two resonances, lying on different sheets, in the energy region between 1300 MeV and 1500 MeV are found. These states may be compared with the resonances $f_0(1370)$ and $f_0(1500)$ seen in the experiments at CERN. Total, elastic and inelastic channel cross sections, branching ratios and coupling constants are evaluated and compared with available data.

1 Model

Scalar mesons are still not well enough identified and classified. Many theoretical and experimental efforts [1, 2] have been recently made for a better understanding of their dynamics and structure. In our analysis we use a separable potential

model [3] for three coupled channels: $\pi\pi$, $K\overline{K}$ and $\sigma\sigma$ (an effective $2\pi 2\pi$ channel). We solve a system of Lippmann-Schwinger equations with relativistic propagators in all channels. The scattering amplitudes, obtained in such a way, allow us to calculate phase shifts and inelasticities as functions of 14 potential parameters. All these parameters are determined from fits to experimental data in the $\pi\pi$ and $K\overline{K}$ channels. The $\pi\pi$ data consist of the $\pi\pi$ phase shifts and inelasticities from the CERN-Cracow-Munich measurements of the $\pi^- p_\uparrow \to \pi^+\pi^- n$ reaction on a polarized target [4]. We furthermore use the $K\overline{K}$ phase shifts obtained from reactions on unpolarized targets [5]. Before fitting we do not assume anything about possible structures of the scattering amplitudes. We find 6 different solutions [3, 6] with χ^2 values smaller than 116 for 102 degrees of freedom. Especially good fits with χ^2 smaller than 101 are obtained for the down-flat set of the $\pi\pi$ data. We then analyse the analytical structure of the calculated scattering amplitudes to find the S-matrix poles and zeroes in the complex energy plane.

In order to understand the role played by interchannel interactions in the scalar meson dynamics we first study the analytical structure of the S-matrix elements in the fully decoupled case where all the interchannel couplings are equal to zero. In such a case all channels are separated and one can recognise in which channel particular poles are created. When the interchannel couplings are switched on, all the poles change positions and split into poles lying on different sheets. The sheets are labelled by (g_1, g_2, g_3) where g_i are the signs of the imaginary parts of the complex momenta k_i in particular channels ($i = 1$ for $\pi\pi$, $i = 2$ for $K\overline{K}$ and $i = 3$ for $\sigma\sigma$).

All diagonal S-matrix elements can be expressed as ratios of two Jost functions $D(k_1, k_2, k_3)$ with minus sign of the momentum k_i in the numerator. Therefore positions of the poles (also in the fully coupled case when the interchannel couplings are switched on) are common for all the S-matrix elements. However, positions of S-matrix zeroes depend on the chosen channel. Only poles and zeroes which lie close enough to the physical region have a significant influence on the phase shifts and inelasticities measured experimentally. We then study poles and zeroes nearby the physical regions and relate them to the scalar resonances.

2 Results

In Table 1 we show the positions of the S-matrix poles for solution A in the fully decoupled case (column "no couplings") and the fully coupled case (column "with couplings"). When the interchannel couplings are switched on, starting from the

Table 1: *Positions of S-matrix poles for the solution A (in MeV)*

channel	no couplings		with couplings		Sign of	
	ReE	ImE	ReE	ImE	$Imk_\pi\ Imk_K\ Imk_\sigma$	No
$\pi\pi$	658	-607	564	-279	$---$	I
			518	-261	$-++$	II
			211	0	$-+-$	III
			532	-315	$--+$	IV
			235	0	$++-$	V
$\pi\pi$	1346	-275	1405	-74	$---$	VI
			1445	-116	$-++$	VII
			1424	-94	$-+-$	VIII
			1456	-47	$--+$	IX
$K\overline{K}$	881	-498	170	0	$+--$	X
			159	0	$---$	XI
			418	-10	$--+$	XII
			1038	-204	$-+-$	XIII
			988	-31	$-++$	XIV
$\sigma\sigma$	118	-2227	4741	-4688	$---$	XV
			3687	-2875	$-+-$	XVI
			3626	-3456	$+--$	XVII
			3533	-579	$++-$	XVIII

Table 2: *Average masses and widths of resonances* $f_0(500)$ *,* $f_0(980)$ *and* $f_0(1400)$ *found in our solutions A, B, E and F. Here errors represent the maximum departure from the average.*

resonance	mass (MeV)	width (MeV)	sheet
$f_0(500)$ or σ	523 ± 12	518 ± 14	$-++$
$f_0(980)$	991 ± 3	71 ± 14	$-++$
$f_0(1400)$	1406 ± 19	160 ± 12	$---$
	1447 ± 27	108 ± 46	$--+$

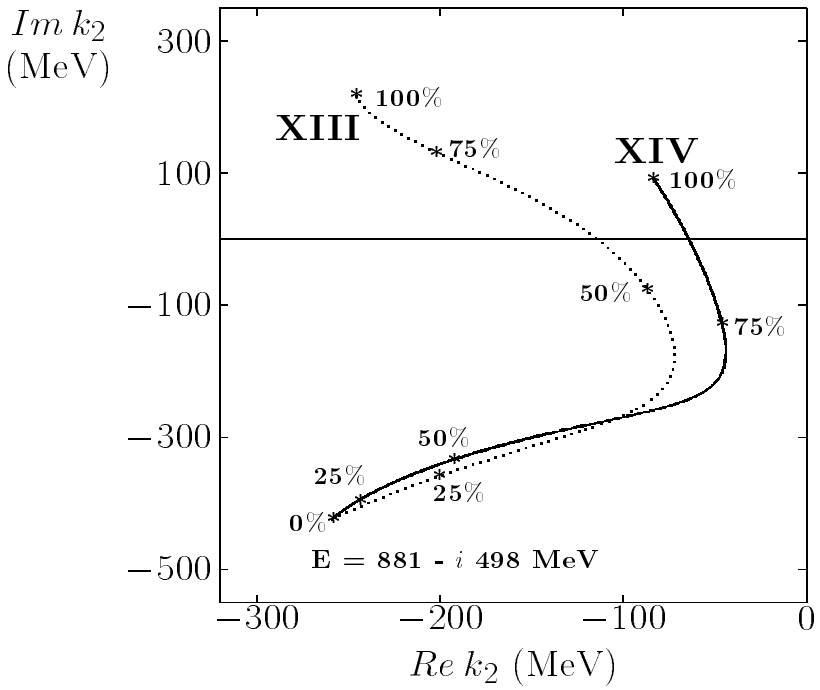

Figure 1: *Pole trajectories in the complex momentum plane as a function of the percentage of the interchannel coupling strength for the solution A in the $K\overline{K}$ channel. Roman numbers and energies are taken from Table 1*

fully decoupled case, the pole in the $K\overline{K}$ channel at $E = 881 - i488$ MeV splits and moves, among others, to two poles XIII and XIV. As can be seen in Fig. 1 both those poles change sheets when the interchannel couplings are increasing. Pole XIV on sheet $(-++)$ lies closer to the physical region than pole XIII on sheet $(-+-)$ and is related to $f_0(980)$ state. For the solutions E and F the corresponding poles lie, in the fully decoupled case, on the positive part of the imaginary axis in the kaon complex momentum plane and therefore they correspond to $K\overline{K}$ bound states. When the interchannel couplings are switched on then these poles move away from the imaginary axis creating so called quasi-bound states. In solutions A, B, C and D state $f_0(980)$ appears as an ordinary resonance in absence of the interchannel interactions. Unambiguous determination of the nature of the $f_0(980)$ state requires new precise experimental data on $K\overline{K}$ interactions near threshold.

In Table 2 we present masses and widths of three scalar resonances found in all our solutions. Particular choice of the sheets for poles is related to positions of both the S-matrix poles and zeroes. An example of such a choice is shown in Fig.2 for different complex momenta planes. We there present positions of the

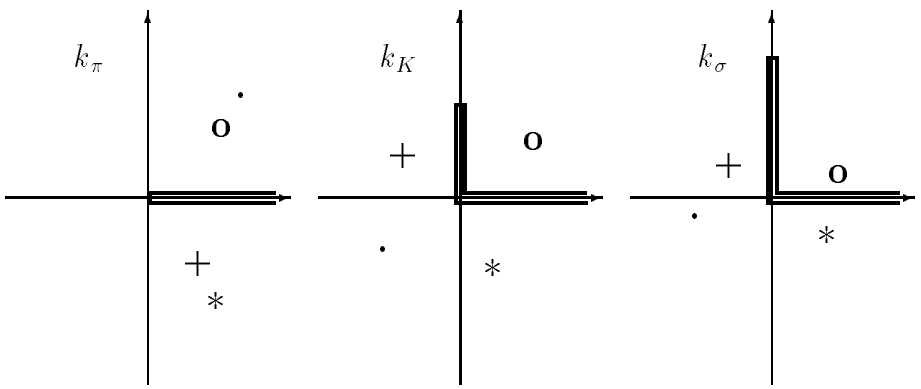

Figure 2: *Planes of complex momenta with schematic positions of the $S_{\pi\pi}$ poles (denoted by $*$ on sheet $---$ and by $+$ on sheet $-++$) and zeroes (denoted by $\cdot$ on sheet $+--$ and by* **o** *on sheet $+++$). Solid double lines denote physical regions in particular channels.*

poles on sheets $(---)$ and $(-++)$ together with the corresponding zeroes on sheets $(+--)$ and $(+++)$ respectively. In the pion complex momentum plane the poles lie below the real axis ($Imk_1 < 0$) close to the physical region. The sheets for the zeroes are determined for the $S_{\pi\pi}$-matrix element. From the relation $S(k_1, k_2, k_3) = S^*(-k_1^*, -k_2^*, -k_3^*)$ we infer that all the poles and zeroes in Fig. 2 have their twins (not indicated in figure) which lie symmetrically with respect to the imaginary axes. Below 1 GeV the poles on sheet $(-++)$ and the corresponding zeroes on sheet $(+++)$ lie close to the physical regions in all channels and are related to the $f_0(500)$ and $f_0(980)$ states.

In vicinity of the $\sigma\sigma$ threshold the two poles on sheets $(---)$ and $(--+)$, with the corresponding zeroes on sheets $(+--)$ and $(+-+)$ respectively, lie close to the physical regions in all channels. We call both of them $f_0(1400)$. Existence of two nearby states in the energy range between 1300 and 1500 MeV was recently considered in many phenomenological works [1)]. In our analysis an appearance of these states in the fully coupled case is due to the splitting of one single pole in the fully decoupled case.

An interesting example of the interplay of the S-matrix poles and zeroes can be found in solution E. In Table 3 we present the set of poles lying close to the energy region 1600–1700 MeV for this solution. All poles come from splitting of the single pole in the $\sigma\sigma$ channel at $E = 1565 - i112$ MeV. As a result of the interchannel interactions the pole XIII and the $S_{\pi\pi}$ zero corresponding to the pole XVI lie on the same sheet $(---)$. As it can be seen in Fig. 3, a flat dependence of the $\pi\pi$

Table 3: *Positions of the S-matrix poles around 1600–1700 MeV for the solution E (in MeV)*

channel	no couplings		with couplings		Sign of	
	ReE	ImE	ReE	ImE	$Imk_\pi\ Imk_K\ Imk_\sigma$	No
$\sigma\sigma$	1565	-112	1703	-271	$---$	XIII
			1648	-67	$-+-$	XIV
			1673	-77	$--+$	XV
			1624	-175	$+--$	XVI

phase shifts above 1600 MeV is the consequence of a strong mutual cancellation of the nearby pole and the zero. In Fig. 4 their influence on the $\sigma\sigma$ scattering amplitude can be, however, noticed in the energy dependence of $\sigma\sigma$ inelasticities. For the solution E the two minima correspond to the states $f_0(1400)$ and $f_0(1700)$. In solutions A and B there is only one minimum related to $f_0(1400)$.

In [3] together with total, elastic and inelastic cross sections we present the branching ratios calculated for the different channels. Above the $\sigma\sigma$ threshold we use 3×3 matrix in which each row describes the three branching ratios in each channel. We find that the phenomenological information contained in such a matrix is much reacher than in the Breit-Wigner approach where only three branching ratios can be used for the three channel case. We perform our calculations in the energy regions near the $f_0(980)$ and $f_0(1400)$ resonances and also for energies between them. We also give formulae for the coupling constants and compare their different approximations. We find that the $f_0(980)$ couples strongly to $K\overline{K}$ and the $f_0(1400)$ to $\pi\pi$ channels. Coupling of the $f_0(1400)$ to the $\sigma\sigma$ channel can be also significant.

3 Conclusions

A proper choice of not only the S-matrix poles but also of its zeroes plays a crucial role in the determination of the scalar meson parameters. It is difficult to find unambiguous values of these parameters looking only at the energy dependence of phase shifts and inelasticities. The physical interpretation becomes especially complicated when the interchannel couplings are strong and lead to large movements of poles and zeroes in the complex energy plane. The positions of both the S-matrix poles and zeroes have a significant influence on the experimentally measured observables. Analysis of the analytical structure of scattering amplitudes is necessary to determine the nature of scalar mesons.

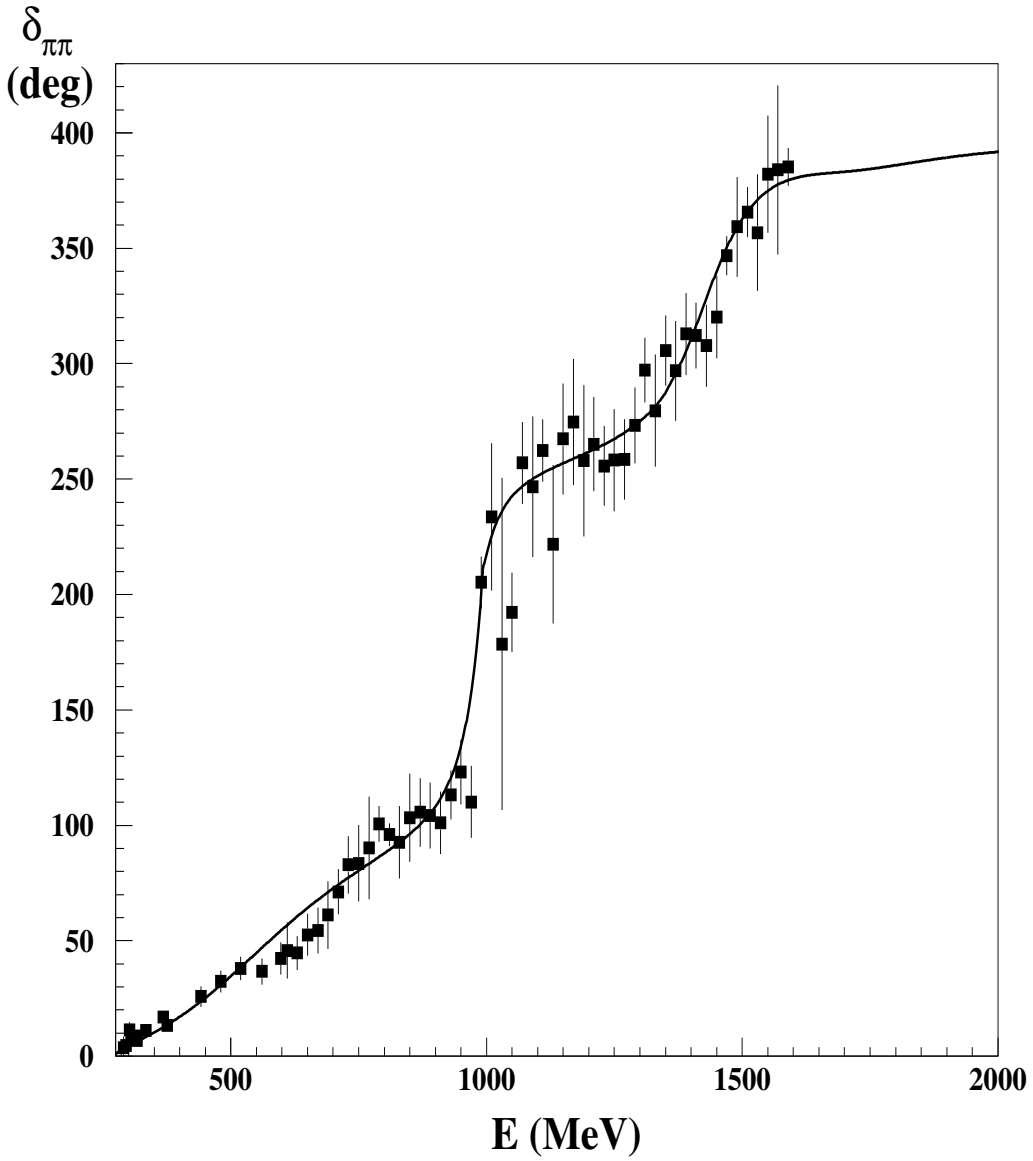

Figure 3: *Energy dependence of $\pi\pi$ phase shifts for the solution E*

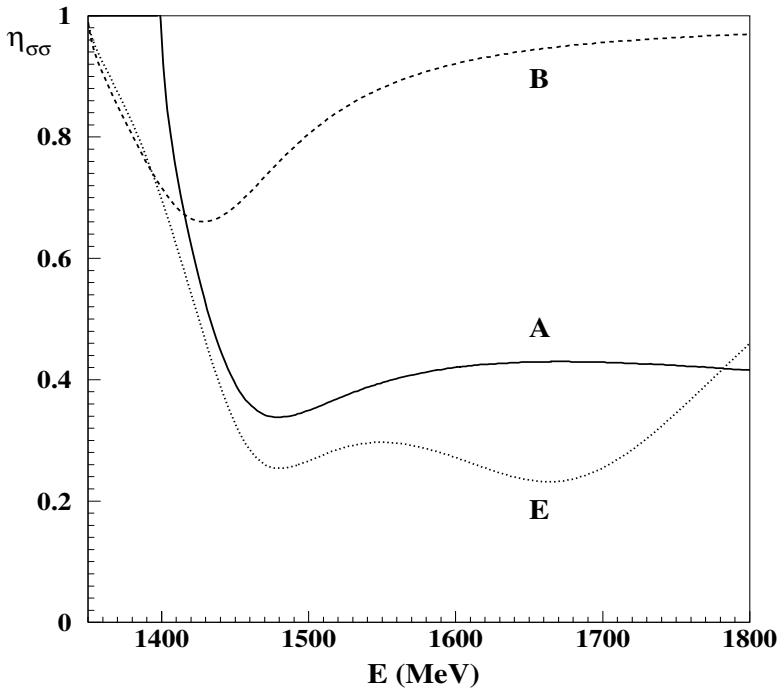

Figure 4: *Energy dependence of the $\sigma\sigma$ inelasticity for the solutions A, B and E*

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