

Glueball Regge trajectories from gauge-string duality and the pomeronHenrique Boschi-Filho,^{1,*} Nelson R. F. Braga,^{1,†} and Hector L. Carrion^{1,2,‡}¹*Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, RJ 21941-972 Brazil*²*Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970, São Paulo-S.P., Brasil*

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The spectrum of light baryons and mesons has been reproduced recently by Brodsky and Teramond from a holographic dual to QCD inspired in the AdS/CFT correspondence. They associate fluctuations about the AdS geometry with four-dimensional angular momenta of the dual QCD states. We use a similar approach to estimate masses of glueball states with different spins and their excitations. We consider Dirichlet and Neumann boundary conditions and find approximate linear Regge trajectories for these glueballs. In particular the Neumann case is consistent with the Pomeron trajectory.

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I. INTRODUCTION

The observation that hadrons show up in approximate linear Regge trajectories was one of the initial motivations for developing string theory. Recently, very good estimates for masses of light mesons and baryons were obtained from string theory in a sliced $\text{AdS}_5 \times S^5$ space-time [1]. On the other side, experimental results for the cross sections of soft processes show an increase with energy corresponding to pomerons with Regge trajectories of the form [2]

$$\alpha(t = M^2) \approx 1.08 + 0.25M^2 \quad (1)$$

where masses are in GeV. Furthermore, it has been suggested that pomerons may be related to glueballs.

We use a similar approach to that of Ref. [1] to estimate masses of glueballs with different angular momenta and obtain the corresponding Regge trajectories. Our results for the glueball trajectories show consistency with that of soft Pomerons.

Strong interactions are well described by QCD [Yang Mills SU(3) plus fermionic matter fields]. In the high energy regime one can perform perturbative calculations. At low energies QCD is nonperturbative and the usual approach is to consider QCD in a lattice. In particular a lattice analysis of the consistency of glueball Regge trajectories with pomerons has been done recently [3].

An alternative approach is to consider a dual description of strong interactions in terms of string theory. A connection between SU(N) gauge theories (for large N) and string theory was realized long ago by 't Hooft [4]. A very important recent result relating gauge and string theories was obtained by Maldacena [5]. He established a correspondence between string theory in $\text{AdS}_5 \times S^5$ space-time and $\mathcal{N} = 4$ Super Yang Mills SU(N) theory for large N in its four dimensional boundary. This super Yang Mills theory is conformal. Soon after, a proposal of a correspondence closer to QCD (nonconformal and nonsupersymmet-

ric) was discussed by Witten [6]. In this formulation QCD would be described by string theory in an AdS-Schwazschild black hole metric. Glueball masses were estimated in this context, using a WKB approximation in [7–14]. Also there are many interesting estimates of glueball masses from dualities involving different geometries generated by string theory, see for example Refs. [15–19].

A phenomenological approach to estimate hadron masses inspired in the AdS/CFT correspondence was proposed in [20,21] and applied to the case of scalar glueballs. An energy scale was introduced in analogy with the discussion of hard scattering from AdS/CFT in [22] (see also [23]). In this approach supergravity fields in an AdS_5 slice times a compact S^5 space are considered as an approximation for a string theory dual to QCD. The metric of this space can be written as

$$ds^2 = \frac{R^2}{z^2} (dz^2 + (d\vec{x})^2 - dt^2) + R^2 d\Omega_5^2, \quad (2)$$

where the size of the slice: $0 \leq z \leq z_{\max}$ is related to the QCD scale

$$z_{\max} = \frac{1}{\Lambda_{\text{QCD}}}. \quad (3)$$

In this phenomenological approach, Dirichlet boundary conditions were imposed at $z = z_{\max}$ and the ratios of the masses of the scalar glueball 0^{++} and its spinless excitations were obtained [20,21]. These results are in good agreement with lattice and AdS-Schwazschild results. Note that we do not consider excitations in the S^5 directions since according to the AdS/CFT correspondence they are related to the supersymmetric structure of the boundary theory. For other results related to strong interactions from AdS/CFT see also [24–35].

II. GLUEBALL MASSES

In Ref. [1] very interesting results for the hadronic spectrum were obtained considering scalar, vector, and fermionic fields in the sliced $\text{AdS}_5 \times S^5$ space. It was proposed that massive bulk states corresponding to fluctu-

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ations about the AdS₅ metric are dual to QCD states with angular momenta (spin) on the four dimensional boundary.

According to the AdS/CFT correspondence, massless scalar string states are dual to boundary scalar glueball operators [36,37]. On the other hand, scalar string excitations with mass μ couple to boundary operators with dimension $\Delta = 2 + \sqrt{4 + (\mu R)^2}$. This happens because these massive states behave as $z^{4-\Delta}$ near the AdS boundary (small z). Scalar glueball operators $\mathcal{O}_4 = F^2$ have dimension 4, while glueballs operators $\mathcal{O}_{4+\ell} = FD_{\{\mu_1 \dots D_{\mu_\ell}\}}F$ with spin ℓ have dimension $4 + \ell$. Then a consistent coupling between string states with mass μ and glueball operators with spin ℓ requires that

$$(\mu R)^2 = \ell(\ell + 4). \quad (4)$$

This means that the masses of these AdS modes have a discrete spectrum since they are in correspondence with glueball operators of integer spin.

We will assume that such dualities established in the AdS/CFT correspondence are approximately valid in our phenomenological model of an AdS slice. So we take a glueball operator with spin ℓ to be dual to massive string states, with mass given by Eq. (4), in the AdS slice.

The glueball operators in the AdS/CFT correspondence are all massless respecting conformal invariance. Once we introduce a size z_{\max} in the AdS space, there will be an infrared cut off in the boundary, which we identify with Λ_{QCD} , breaking conformal invariance. The presence of the slice implies an infinite tower of discrete modes in the z direction for the bulk states. This discretization does not alter the asymptotic behavior (small z) of bulk modes which is related to their mass μ . We assume that these bulk discrete modes in the z direction are related to the masses of the nonconformal glueball operators. Using this model we will calculate glueball masses and the corresponding Regge trajectories.

The solutions for scalar fields with mass μ in AdS₅ satisfy [36,37]

$$\left[z^3 \partial_z \frac{1}{z^3} \partial_z + \eta^{\mu\nu} \partial_\mu \partial_\nu - \frac{(\mu R)^2}{z^2} \right] \phi = 0. \quad (5)$$

Considering plane wave solutions in the four dimensional coordinates \vec{x} and t for states with mass μ given by Eq. (4) one can write the solutions as

$$\phi(x, z) = C_{\nu,k} e^{-iP \cdot x} z^2 J_\nu(u_{\nu,k} z), \quad (6)$$

where $\nu = 2 + \ell$ and the discrete modes $u_{\nu,k}$ ($k = 1, 2, \dots$) are determined by the boundary conditions. Here we will consider two possibilities:

$$\phi(z = z_{\max}) = 0 \quad (\text{Dirichlet}); \quad (7)$$

$$\partial_z \phi|_{z=z_{\max}} = 0 \quad (\text{Neumann}). \quad (8)$$

TABLE I. Masses of glueball states J^{PC} with even J expressed in GeV, estimated using the sliced AdS₅ \times S⁵ space with Dirichlet boundary conditions. The mass of 0^{++} is an input from lattice results [38,39].

Dirichlet glueballs	lightest state	1st excited state	2nd excited state
0^{++}	1.63	2.67	3.69
2^{++}	2.41	3.51	4.56
4^{++}	3.15	4.31	5.40
6^{++}	3.88	5.85	6.21
8^{++}	4.59	5.85	7.00
10^{++}	5.30	6.60	7.77

A. Dirichlet boundary conditions

In the case of Dirichlet boundary conditions, as used in Refs. [1,20,21], one obtains

$$u_{\nu,k} = \frac{\chi_{\nu,k}}{z_{\max}} = \chi_{\nu,k} \Lambda_{\text{QCD}}; \quad J_\nu(\chi_{\nu,k}) = 0. \quad (9)$$

Assuming the duality between these modes in the AdS₅ slice and the glueball operators, the scalar glueball 0^{++} is related to the massless scalar. So its mass is proportional to $\chi_{2,1}$. The excited scalar glueball states $0^{++* \dots}$ correspond to the other values of k and their masses are proportional to $\chi_{2,k}$.

The higher angular momenta glueballs J^{++} are related to the massive scalar modes according to $(\mu R)^2 = \ell(\ell + 4)$ with $\ell = J$. Then the mass of the lightest state with angular momentum J is proportional to $\chi_{2+\ell,1}$. The corresponding excitations are proportional to $\chi_{2+\ell,k}$.

We show in Table I the results for even angular momenta that may be related to the phenomenological pomeron which has a trajectory with even signature. We introduced the mass of the lightest glueball as an input and found the glueball spectrum from Eq. (9). This input for the lightest glueball is in accordance with lattice results [38,39]. The results for the excitations of 0^{++} were obtained previously in [20,21] and are in good agreement with the masses estimated using AdS-Schwarzschild black hole metric [7–10,12–14].

Our result for the ratio of masses $M_{2^{++}}/M_{0^{++}} = 1.48$ is in good agreement with lattice [38,39] and deformed conifold results [17].

B. Neumann boundary conditions

Considering Neumann boundary conditions, the vanishing of the scalar field derivative at z_{\max} leads to

$$(2 - \nu)J_\nu(\xi_{\nu,k}) + \xi_{\nu,k}J_{\nu-1}(\xi_{\nu,k}) = 0. \quad (10)$$

The correspondence between QCD and scalar string states is taken exactly as in the Dirichlet case. The glueball masses are now given by

$$u_{\nu,k} = \frac{\xi_{\nu,k}}{z_{\max}} = \xi_{\nu,k} \Lambda_{\text{QCD}} \quad (11)$$

TABLE II. Masses of glueball states J^{PC} with even J expressed in GeV, estimated using the sliced $\text{AdS}_5 \times S^5$ space with Neumann boundary conditions. The mass of 0^{++} is an input from lattice results [38,39].

Neumann glueballs	lightest state	1st excited state	2nd excited state
0^{++}	1.63	2.98	4.33
2^{++}	2.54	4.06	5.47
4^{++}	3.45	5.09	6.56
6^{++}	4.34	6.09	7.62
8^{++}	5.23	7.08	8.66
10^{++}	6.12	8.05	9.68

In this case we also take the mass of the lightest glueball as an input. The results for states with even spin are shown in Table II.

Here in the Neumann case the ratios of the masses

$$\frac{M_{2^{++}}}{M_{0^{++}}} = 1.56 \quad (12)$$

$$\frac{M_{0^{+++}}}{M_{0^{++}}} = 1.83 \quad (13)$$

are in very good agreement with lattice [38,39] and deformed conifold results [17].

III. REGGE TRAJECTORIES

From the results of Tables I and II one finds relations between spin and mass squared for the glueballs that represent the corresponding Regge trajectories. We find that these trajectories for the glueballs are nonlinear. This is in agreement with general properties of Regge trajectories, as discussed for example in [40].

In order to compare these results with the Pomeron behavior of Eq. (1) it is interesting to consider linear approximations for these trajectories as in Ref. [2]

$$J = \alpha(t = M^2) = \alpha_0 + \alpha' M^2. \quad (14)$$

Here we will be interested in the trajectories of the glueball lightest states with even J only. Also, as discussed in [3] the 0^{++} glueball is not expected to contribute to the Pomeron trajectory that has a positive intercept. For more discussions on the Pomeron intercept see, for instance, [41]. So we will consider linear fits for the states $2^{++}, 4^{++}, \dots$ for both Dirichlet and Neumann boundary conditions.

In particular, for the Neumann case the results are compatible with the Pomeron trajectory. For instance, for the states J^{++} with $J = 2, 4, \dots, 10$ we find

$$\alpha' = (0.26 \pm 0.02) \text{ GeV}^{-2}; \quad \alpha_0 = 0.80 \pm 0.40 \quad (15)$$

This trajectory is shown in Fig. 1. Note that we are not considering errors in the masses of the Glueballs. The

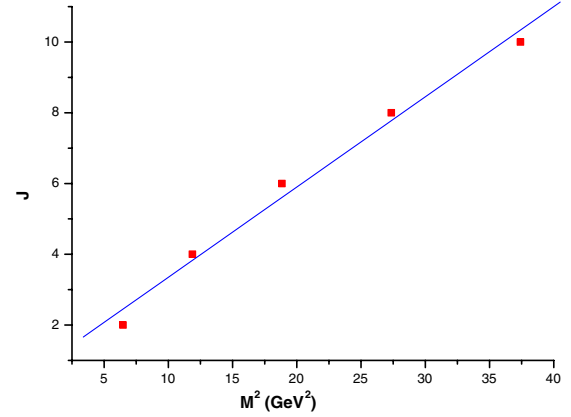


FIG. 1 (color online). Approximate linear Regge trajectory for Neumann Boundary condition for the states $2^{++}, 4^{++}, 6^{++}, 8^{++}, 10^{++}$.

errors appearing in the estimated coefficients α' and α_0 refer to the deviations with respect to the linear fit.

For other set of points we also find results compatible with the Pomeron trajectory. In particular for the set of states $4^{++}, 6^{++}, 8^{++}$ we find $\alpha' = (0.26 \pm 0.01) \text{ GeV}^{-2}$ and $\alpha_0 = 1.01 \pm 0.30$.

We note that the slope α' in the Neumann case does not vary considerably with the set of states considered in the linear approximation. The error of the slope is still small and consistent with the Pomeron result.

For the Dirichlet case, taking the states J^{++} with $J = 2, 4, \dots, 10$, we find a linear fit with

$$\alpha' = (0.36 \pm 0.02) \text{ GeV}^{-2}; \quad \alpha_0 = 0.32 \pm 0.36. \quad (16)$$

These states and the corresponding linear fit are shown in Fig. 2. The slope of this trajectory is higher than the Pomeron result in Eq. (1). For other sets of states using Dirichlet boundary condition we also find linear approx-

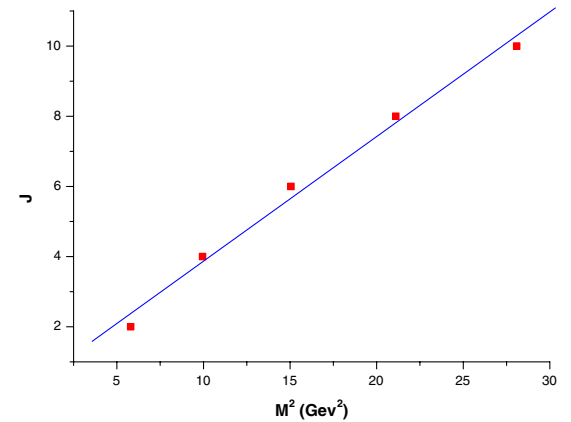


FIG. 2 (color online). Approximate linear Regge trajectory for Dirichlet Boundary condition for the states $2^{++}, 4^{++}, 6^{++}, 8^{++}, 10^{++}$.

imations with slopes which are higher than that of the Pomeron. For instance, with $J = 4, \dots, 10$ we find $\alpha' = (0.33 \pm 0.02) \text{ GeV}^{-2}$ and $\alpha_0 = 0.90 \pm 0.32$.

IV. CONCLUSION

We found simple estimates for masses of glueballs of different spins in a sliced $\text{AdS}_5 \times S^5$ inspired in the AdS/CFT duality. These results are in good agreement with other estimates in the literature. It is remarkable that for the case of Neumann boundary condition the linear approximation for glueball Regge trajectories

$$\alpha(t = M^2) = (0.80 \pm 0.40) + (0.26 \pm 0.02)M^2 \quad (17)$$

is consistent with the Pomeron trajectory of Eq. (1).

This result shows that the Neumann boundary condition seems to work better than Dirichlet for glueballs in this holographic model. Both choices correspond to vanishing flux for bulk scalar fields at $z = z_{\text{max}}$ and would be physically acceptable conditions. It is interesting to note that similar Neumann conditions appear in the Randall Sundrum model [42] as a consequence of the orbifold condition.

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