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The low-lying glueball spectrum

Adam P. Szczepaniak [a], Eric S. Swanson [b,c]

[a] Department of Physics and Nuclear Theory Center, Indiana University, Bloomington, IN 47405, USA
[b] Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA
[c] Jefferson Lab, 12000 Jefferson Ave, Newport News, VA 23606, USA

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Abstract

The complete low-lying positive charge conjugation glueball spectrum is obtained from QCD. The formalism relies on the construction of an efficient quasiparticle gluon basis for Hamiltonian QCD in Coulomb gauge. The resulting rapidly convergent Fock space expansion is exploited to derive quenched low-lying glueball masses with no free parameters which are in remarkable agreement with lattice gauge theory.

1. Introduction

The scalar glueball has been called the fundamental particle of QCD [1]. Indeed, its existence and nonzero mass are a direct consequence of the non-Abelian nature of QCD and the confinement phenomenon. It is clear that finding and understanding the scalar (and other) glueballs is a vital step in mastering low-energy QCD.

Recently quenched lattice gauge theory has been able to determine the low-lying glueball spectrum with reasonable accuracy [2] (only very preliminary determinations of other matrix elements have been attempted). These data serve as a useful benchmark in the development of a qualitative model of the emergent properties of low-energy QCD. The models may then be used to guide experimental glueball searches.

Previous models of glueballs have relied on ad hoc effective degrees of freedom such as flux tubes [3], bags [4], or constituent gluons [5]. We note that some of the models listed in Ref. [5] construct states with massive gluons, while others either use transverse gluons or dynamically generated gluons masses. Models in the former category contain spurious states due to the presence of unphysical longitudinal gluon modes. Sum rule computations of glueball properties exist [6], however, they are based on phenomenological properties of the spectrum. Finally, the conjectured duality between supergravity and large-N gauge theories has been used to compute the glueball spectrum in non-supersymmetric Yang–Mills theory by solving the supergravity wave equations in a black hole geometry [7]. Unfortunately all of these approaches suffer from weak or conjectured connections to QCD.

We present a computation of the positive charge conjugation glueball spectrum which arises from QCD and is systematically improvable. The computation

E-mail address: aszczepa@indiana.edu (A.P. Szczepaniak).

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is based on the formalism presented in Ref. [8] in which the QCD Hamiltonian in Coulomb gauge is employed as a starting point. Coulomb gauge is efficacious for the study of bound states because all degrees of freedom are physical (there are no ghost fields in this gauge) and a positive definite norm exists [9]. Furthermore, resolving the Coulomb gauge constraint produces an instantaneous interaction (the non-Abelian analogue of the Coulomb interaction) which, as shown in Ref. [8] may be used to generate bound states. Because the temporal component of the vector potential is renormalization group invariant in Coulomb gauge (this is not true in other gauges), the instantaneous potential does not depend on the ultraviolet regulator or the renormalization scale [12]. This fact permits a physical interpretation of the instantaneous potential which is a central aspect of our formalism.

The pure gauge QCD Hamiltonian may be written as [9] $H_{\rm QCD} = H_0 + \delta H$ with

$$H_0 = \frac{1}{2}\int d\mathbf{x}\left[\boldsymbol{E}^2 + \boldsymbol{B}^2\right] + \frac{1}{2}\int d\mathbf{x}\, d\mathbf{y}\, \rho^a(\mathbf{x}) K^{(0)}(\mathbf{x}-\mathbf{y})\rho^a(\mathbf{y}) \quad (1)$$

and

$$\delta H = V_{3g} + V_{4g} + V_J + V_C. \quad (2)$$

Here $\boldsymbol{B} = \nabla \times \mathbf{A}$ is the Abelian part of the chromomagnetic field and $\boldsymbol{E} = -\partial/\partial t\mathbf{A}$ is the chromoelectric field. The third term in H_0 represents the non-Abelian, instantaneous Coulomb interaction between color charges, $\rho^a = -f^{abc}\mathbf{E}^b \cdot \mathbf{A}^c$, mediated by an effective potential K^0 computed by taking a vacuum expectation value of the Coulomb kernel,

$$K^0(\mathbf{x}-\mathbf{y})\delta_{ab} = g^2\langle\Psi|\left[(\boldsymbol{\nabla}\cdot\boldsymbol{D})^{-1}\left(-\nabla^2\right)(\boldsymbol{\nabla}\cdot\boldsymbol{D})^{-1}\right]_{\mathbf{x},a;\mathbf{y},b}|\Psi\rangle, \quad (3)$$

with $\boldsymbol{D}^{ab} = \delta^{ab}\boldsymbol{\nabla} - gf^{abc}\mathbf{A}^c$ being the covariant derivative in the adjoint representation. For the vacuum wave functional, $\Psi[\boldsymbol{A}] = \langle \boldsymbol{A}|\Psi\rangle$ we take a variational ansatz,

$$\Psi[\boldsymbol{A}] = \exp\left(-\frac{1}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\, \mathbf{A}^a(\mathbf{k})\omega(k)\mathbf{A}^a(-\mathbf{k})\right), \quad (4)$$

with the variational parameter $\omega(k)$ determined by minimizing the vev of H. The correction terms in δH include V_{3g} and V_{4g} which are the three- and four-gluon operators originating from the difference between the full and the Abelian chromomagnetic field. V_J denotes a contribution from the Faddeev–Popov determinant in the kinetic term. The effects of V_J and the Faddeev–Popov determinant in the functional integrals have recently being studied in Ref. [10] where it was found that its effects can be effectively included in the variational parameter $\omega(k)$. Finally, V_C is the difference between the Coulomb operator and its vev, K^0. In the calculation of the glueball spectrum it results in operators mixing two and three, quasiparticle wave functions. We note that after renormalization the coupling g is absorbed into the Faddeev–Popov operator, which then defines the Coulomb gauge analog of a ghost propagator [8, 10]. The renormalized effective potential K^0 is fixed by comparing with the quenched lattice QCD static potential. A very accurate representation of the static confinement potential is achieved [8].

The variational vacuum defined above also specifies a Fock space of quasiparticle excitations corresponding to effective gluonic degrees of freedom (which we call quasigluons). Such quasigluons obey "massive" dispersion relation in the variational vacuum and therefore improve the description of gluonic bound states since mixing between states with different number of quasiparticles is suppressed due to their effective mass.

The calculation of the vev of the Hamiltonian and the properties of the quasiparticle excitations were discussed in Refs. [8,10,11]. These require solving a set of coupled integral Dyson equations and as a result one finds that the function $\omega(k)$, which in a free theory is given by $\omega(k) = k$, becomes finite as $k \to 0$. The value $\omega(0)$ can be related to the slope of the static potential at large distances.

2. Fock space expansion and the glueball spectrum

The quasigluons which emerge in the analysis of Ref. [8] set the QCD scale parameter via the low momentum dispersion relation $r_0\omega(k \to 0) = 1.4$, where r_0 is the lattice Sommer parameter. Using the

Regge string tension or ρ mass to fix the scale then gives $\omega(0) = 600$–650 MeV. It is natural to interpret this scale as a dynamical gluon mass.[1] Thus the formalism of Ref. [8] provides a justification of a Fock space expansion in terms of quasigluons and gives the leading instantaneous interaction between the quasigluons. In view of this it is natural to attempt a description of low-lying glueballs in the pure gauge sector of QCD.

In this approach positive charge conjugation glueballs are dominated by the two quasigluon contribution. These may mix with three and higher quasigluon states via transverse gluon exchange (and, in general, via any term in δH). Mixing with single quasigluon states is excluded because color nonsinglet states are removed from the spectrum due to infrared divergences in the color nonsinglet spectrum [11]. Finally, the scalar glueball is orthogonal to the vacuum due to the form of the gap equation.

The resulting bound state equations are shown in Eq. (5). There is one orbital component of glueball wave function for $J^P = 0^+$ and two for $J^P = (\text{even} \geqslant 2)^+$. These are denoted by $\psi_i(k)$, $i = 1, 2$. The first term on the right-hand side of Eq. (5) represents the quasigluon kinetic energy (the gluon gap equation has been employed to simplify the expression), the second term is the quasigluon self energy, and third represents the interaction between quasigluons in the channel of interest.

$$\begin{aligned}
&\int \frac{k^2\,dk}{(2\pi)^3}\, 2\omega(k)|\psi_i(k)|^2 \\
&\quad + \frac{N_C}{2}\sum_i \int \frac{k^2\,dk}{(2\pi)^3}\,\frac{q^2\,dq}{(2\pi)^3}\,\frac{\omega(k)}{\omega(q)} \\
&\qquad\qquad \times \left[\frac{4}{3}V_0 + \frac{2}{3}V_2\right]|\psi_i(k)|^2 \\
&\quad - \frac{N_C}{4}\int \frac{k^2\,dk}{(2\pi)^3}\,\frac{q^2\,dq}{(2\pi)^3}\,\frac{(\omega(k)+\omega(q))^2}{\omega(k)\omega(q)} \\
&\qquad \times \psi_i^*(q) K_{ij}(q,k)\psi_j(k) \\
&\quad = E\int \frac{k^2\,dk}{(2\pi)^3}\,|\psi_i(k)|^2, \qquad (5)
\end{aligned}$$

with

$$\begin{aligned}
K_{11} &= \frac{3J^2+3J-2}{(2J-1)(2J+3)}V_J \\
&\quad + \frac{J(J-1)}{2(2J-1)(2J+1)}V_{J-2} \\
&\quad + \frac{(J+1)(J+2)}{2(2J+3)(2J+1)}V_{J+2}, \qquad (6)
\end{aligned}$$

$$\begin{aligned}
K_{22} &= \frac{3(J+2)(J-1)}{(2J-1)(2J+3)}V_J \\
&\quad + \frac{(J+2)(J+1)}{2(2J+1)(2J-1)}V_{J-2} \\
&\quad + \frac{J(J-1)}{2(2J+1)(2J+3)}V_{J+2} \qquad (7)
\end{aligned}$$

and

$$\begin{aligned}
K_{12} &= K_{21} \\
&= \sqrt{(J-1)J(J+1)(J+2)} \\
&\quad \times \left[\frac{1}{2(2J+3)(2J+1)}V_{J+2} \right.\\
&\qquad + \frac{1}{2(2J+1)(2J-1)}V_{J-2} \\
&\qquad \left. - \frac{1}{(2J+3)(2J-1)}V_J\right]. \qquad (8)
\end{aligned}$$

The bound state equations for other glueballs are as in Eq. (5), with the exception that the wave function index takes on a single value. For these cases the interaction kernels are as follows:
$J^P = (\text{odd} \geqslant 3)^+$ (there is no 1^+ gg glueball):

$$K = \frac{J+2}{2J+1}V_{J-1} + \frac{J-1}{2J+1}V_{J+1}; \qquad (9)$$

$J^P = (\text{even} \geqslant 0)^-$:

$$K = \frac{J}{2J+1}V_{J-1} + \frac{J+1}{2J+1}V_{J+1}; \qquad (10)$$

$J^P = 0^+$:

$$K = \frac{2}{3}\left(V_0 + \frac{V_2}{2}\right). \qquad (11)$$

In all these relations the interaction is defined as

$$V_L(q,k) = 2\pi \int_{-1}^{1} dx\, K^{(0)}(q,k,x = \hat{\mathbf{k}}\cdot\hat{\mathbf{q}}) P_L(x) \qquad (12)$$

[1] The relationship of a dynamical gluon mass to vortex-driven confinement and gauge symmetry breaking is thoroughly discussed by Cornwall; see, for example, the second and third references of [5].

and the potential $K^{(0)}(q,k,x)$ is that derived in Ref. [8] with the QCD scale chosen to be $\omega(0) = 600$ MeV. Finally we note that there are no $J^P = (\text{odd})^-$, or $C = -$ glueballs at lowest order in the Fock space expansion.

3. Higher order corrections

It is of course desirable to test the efficacy of the Fock space expansion employed here by explicitly checking the effect of coupling to the three or higher quasigluon spectrum. This is a difficult coupled channel problem and we therefore content ourselves with a perturbative evaluation of these effects in this initial study. Specifically, the energy shift $\delta E_n = \sum_m |\langle gg|\delta H|m(ggg)\rangle|^2/(E_n - E_m)$ must be evaluated. Duality implies that when the energy transfer is large, $(E_n - E_m) > \Lambda$, where Λ is of the order of the QCD scale, this sum may be evaluated in its perturbative form (with partonic gluons in the intermediate state). We compute here the effects of the three-gluon coupling from δH. This is the leading interaction in terms of expansion in the coupling constant. After renormalization $g^2/4\pi \to \alpha(p^2)$ where p^2 represents a characteristic momentum in integrals when computing matrix elements. The running coupling expansion for the remaining sum over low energy modes is certainly less justifiable, however as shown in examples in Ref. [8], such soft corrections also seem to be small. For numerical efficiency the low energy part of the guasigluon exchange was approximated with a local four-gluon interaction (we note that this effective interaction also accounts for the four-gluon interaction present in the Hamiltonian)

$$\begin{aligned} V_c = C(\Lambda) f^{abc} f^{ade} \int & \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{d^3k_4}{(2\pi)^3} \\ & \times (2\pi)^3 \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4) \\ & \times \exp\bigl(-\bigl(k_1^2+k_2^2+k_3^2+k_4^2\bigr)/\Lambda^2\bigr) \\ & \times A_i^b(\mathbf{k}_1) A_j^c(\mathbf{k}_2) A_i^d(\mathbf{k}_3) A_j^e(\mathbf{k}_4), \end{aligned} \tag{13}$$

where C is a dimensionless parameter of the order of $g^2(\Lambda) \sim 1$. Standard effective field theory techniques were subsequently employed: the factorization scale Λ was chosen and the coupling C was fixed by comparison to the lattice scalar glueball mass. Other mass predictions then follow. The scale Λ was then varied to ensure that the procedure is stable and that the coupling remains “natural” (of order unity). To maintain consistency the effect of these terms should be incorporated into the quasigluon gap equation and the gluon self energy. However, it may be shown that the effect of contact terms are canceled in the bound state equation when the gap equation is used to simplify the quasigluon kinetic and self energies. This is not true for the high-momentum gluon exchange terms which add a UV dominated correction to the single gluon kinetic energy. These have negligible effect on low energy spectrum and have been ignored.

4. Results and conclusions

The lowest order predictions for the quenched positive charge conjugation glueball spectrum are presented in Table 1. We stress that there are no free parameters in this computation; the renormalization group parameters and the scale were fixed by comparison to the Wilson loop static interaction [8]. Although one may anticipate splittings on the order of 100 MeV due to coupled channel effects, the general agreement with lattice data is quite good (the χ^2 per degree of freedom for the six measured lowest spin-parity states is 1.5). Nevertheless we note that the authors of Ref. [2] state that the 3^{++} may have significant mixing with higher states and the quoted 4^{++} mass should be regarded as preliminary.

Although it appears to be difficult to push lattice mass computations above 4 GeV it would be interesting to measure the quenched 4^{-+} glueball mass to test the prediction made in Table 1. Lastly we note that all radial excitations lie roughly 1 GeV above their respective ground states, except the 4^{++}. We have no explanation for this curiosity, but note that it implies that lattice extractions of the 4^{++} mass must be made with great care.

We note that the degeneracy between parity states reported in the first reference of [5] is not seen here. We suspect that the degeneracy is due to the nonrelativistic expansion of the interaction kernel made in that reference.

As stated above, coupled channel effects are expected to modify the spectrum at the 10–100 MeV level. As an initial estimate of the size of these effects we simply set $C = 0$ (Eq. (13)) and evaluate the energy

Table 1
Glueball spectrum

State	This work (no mixing)	LGT (GeV)	Ref.
0^{++}	1.98	1.73(5)(8)	[2][a]
		1.754(85)(86)	[15]
		1.627(83)	[16]
		1.686(24)(10)	[19]
		1.645(50)	[18]
		1.61(7)(13)	[17]
$0^{++\prime}$	3.26	2.67(18)(13)	[2]
2^{++}	2.42	2.40(2.5)(12)	[2]
		2.417(56)(117)	[15]
		2.354(95)	[16]
		2.26(12)(18)	[17]
		2.380(67)(14)	[19]
		2.337(100)	[18]
$2^{++\prime}$	3.11	3.499(43)(35)	[13]
0^{-+}	2.22	2.59(4)(13)	[2]
		2.19(26)(18)	[17]
$0^{-+\prime}$	3.43	3.64(6)(18)	[2]
2^{-+}	3.09	3.10(3)(15)	[2]
$2^{-+\prime}$	4.13	3.89(4)(19)	[2]
3^{++}	3.33	3.69(4)(18)[b]	[2]
$3^{++\prime}$	4.29		
4^{++}	3.99	3.65(6)(18)	[14]
$4^{++\prime}$	4.28		
4^{-+}	4.27		
$4^{-+\prime}$	4.98		

[a] The first error is combined statistical and systematic, the second is from scale fixing.

[b] Possible mixing with higher states.

shift due to perturbative one-gluon exchange. As expected, the scalar glueball mass experiences the largest shift, with a reduction in mass of roughly 200 MeV (from 1980 to 1790 MeV). It is gratifying that this brings the scalar glueball into excellent agreement with lattice gauge theory. We proceed by incorporating the effective contact interaction. The factorization scale was varied between 0.25 and 10 GeV, stable results were found between 1 and 8 GeV, with a value of C given roughly by -0.4 in this range. We find that the tensor glueball mass is reduced by roughly 100 MeV, while other masses experience somewhat smaller shifts. Thus it appears that low-lying glueballs are indeed dominated by their two-quasigluon Fock components. However it is clear that a careful examination of coupled channel effects and better lattice data are required to make a definite statement about the efficacy of our approach.

Fluctuations of the topological charge density have pseudoscalar quantum numbers. This raises the possibility that the QCD anomaly affects the lightest 0^{-+} glueball mass. Topological effects have so far not been incorporated into our formalism. Doing so would require modification of the vacuum ansatz to reflect the identification of gauge equivalent field configurations at the boundary of the fundamental modular region. This allows contributions from field configurations with nonzero topological charge. Indeed a cross-over between the 0^{-+} and 2^{++} glueball masses has been observed on the lattice as a function of the renormalized coupling [20] if boundary conditions are imposed.

Further aspects of the gluonic structure laid out in Ref. [8] may be investigated by an examination of the adiabatic potential surfaces of heavy quark hybrids (this probes nonperturbative gluon-confinement potential couplings). Extensions to the light hybrid spectrum will prove of interest to searches for these new states at Jefferson Lab. Finally, the short range structure of the meson sector is dominated by coupled channel effects and nonperturbative gluodynamics. The wealth of experimental information in this sector will provide a definitive test of the dynamics being advocated in our approach.

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